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Module Overview

The Teaching Mathematics TEKS through Technology Professional development is designed to provide teachers an opportunity to increase their depth of understanding about the judicious use of technology in the mathematics classroom. Expected learning outcomes for participants include an understanding of how technology can:

- Provide access to a deeper understanding of mathematical content;
- Provide access to "real world" mathematical topics;
- Improve the economy and efficiency of teaching mathematics TEKS relative to time;
- Facilitate the use of various instructional tools in a mathematical setting.

The structure of the professional development will be designed around the inquiry based 5E instructional model. This model has a strong foundation in research and has been shown to be highly effective in instructional settings.

The components of the "5E" Instructional Model are:

ENGAGE:
The instructor initiates this phase by asking well-chosen questions, posing a problem to be solved, or showing something intriguing. The activity should be designed to interest participants in the problem and to make connections between past and present learning.

The goal of the Engage phase is to begin conversations about data. As participants see the value of data and the mathematics that can be explored and reinforced through the use of data, they will begin to seek data. Technology offers the tools to make sense of data efficiently. Technology also offers effective means for representing data so that analysis may take place. Participants work with data from the Internet, an almanac, data collection devices, and basic measuring tools. They compare the different methods and determine similarities and differences as well as the benefits of each method.

The presenter's role is to ask well-chosen questions to guide the activity but allow participants to proceed in a nonjudgmental fashion. These questions are provided in the leader notes of the training.

EXPLORE/EXPLAIN:
Explore
The exploration phase provides the opportunity for participants to become directly involved with the key concepts of the lesson through guided exploration that requires them to probe, inquire, and question. As we learn, the puzzle pieces (ideas and concepts necessary to solve the problem) begin to fit together or have to be broken down and reconstructed several times. In this phase, presenters observe and listen to participants as they interact with each other and the activity. Presenters ask probing questions to help participants clarify their understanding of major concepts and redirect the participants when necessary.

Explain
In the explanation phase, collaborative learning teams begin to logically sequence events and facts from the investigation and communicate these findings to each other and the presenter. The presenter, acting in a facilitation role, uses this phase to offer further explanation and provide additional meaning or information, such as formalizing correct terminology. Giving labels or correct terminology is far more meaningful and helpful in retention if it is done after the learner has had a direct experience. The explanation phase is used to record the learner's development and grasp of the key ideas and concepts of the lesson.

There are 3 Explore/Explain cycles in this module.

In the first Explore/Explain cycle, participants roll a marble down a ramp and collect data to describe the location of the marble along its projectile path at any given moment in time. Participants then use this model to predict (using a variety of methods) where to locate a cup on a stack of textbooks in order for the marble to roll down the ramp then land inside the cup.

In the second Explore/Explain cycle, participants collect exponential data using a Geometer's Sketchpad sketch containing a sequence of golden triangles. They then analyze the data using the calculator, spreadsheets, and TI-Interactive. This cycle also demonstrates to participants how geometry can be used as a context to explore Algebra 2 functions.

In the third Explore/Explain cycle, participants collect light intensity data using a CBL and a light sensor. Participants then generate a model using an inverse-square parent function.

The presenter's role in the Explore/Explain phases is to ask well-chosen questions to guide participants and clarify their understandings. These questions are provided in the leader notes of the training.

ELABORATE:
The elaboration phase allows for participants to extend and expand what they have learned in the first three phases and connect this knowledge with their prior learning to create understanding. It is critical that presenter verify participants' understanding during this phase.

In the elaborate phase a problem is posed to the participants. Participants are given a simplified form of the 1960 University of Illinois "Doomsday" population model in which it is predicted that the Earth's population will exceed its resources in 2026. Participants collect population data since 1960 to verify the accuracy of the model then use population data to construct a more accurate model.

The presenter's role in the Elaborate phase is to ask well-chosen questions to guide participants' and extend their understandings. These questions are provided in the leader notes of the training.

EVALUATE:
Throughout the learning experience, the ongoing process of evaluation allows the instructor to determine whether or not the participant has reached the desired level of understanding of the key ideas and concepts. More formal evaluation can be conducted at this phase.

Participants will review the instructional phases of this professional development and the classroom-ready lessons according to the list of attributes generated in the elaborate phase of the professional development. Revisions to the list of attributes may occur. Participants will engage in discussion about how each lesson exhibits a judicious use of technology; i.e., participants will address the question, “How does the use of technology in this student lesson help me teach the concepts and skills more effectively and efficiently?”

The presenter’s role in the Evaluate phase is to ask well-chosen questions to assess participants’ understandings as they evaluate student lessons for judicious use of technology. These questions are provided in the leader notes of the training.

STUDENT LESSONS

This training is specifically designed for adult learners. Student lessons with detailed teacher notes and resources are provided to facilitate the implementation of the knowledge acquired by teachers in the professional development.

TABLE OF CONTENTS

Algebra 2

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Algebraic thinking and symbolic reasoning. Symbolic reasoning plays a critical role in algebra; symbols provide powerful ways to represent mathematical situations and to express generalizations. Students study algebraic concepts and the relationships among them to better understand the structure of algebra.

(3) Functions, equations, and their relationship. The study of functions, equations, and their relationship is central to all of mathematics. Students perceive functions and equations as means for analyzing and understanding a broad variety of relationships and as a useful tool for expressing generalizations.

(4) Relationship between algebra and geometry. Equations and functions are algebraic tools that can be used to represent geometric curves and figures; similarly, geometric figures can illustrate algebraic relationships. Students perceive the connections between algebra and geometry and use the tools of one to help solve problems in the other.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

Foundations: Processes	(2A.1) **Foundations for functions.** The student uses properties and attributes of functions and applies functions to problem situations.	The student is expected to: (A) identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations; and (B) collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.
Foundations: Tools	(2A.2) **Foundations for functions.** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.	The student is expected to: (A) use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations; and (B) use complex numbers to describe the solutions of quadratic equations.

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Systems	(2A.3) **Foundations for functions.** The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situations.	The student is expected to: (A) analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems; (B) use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities; and (C) interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.
Parent Functions and Transformations	(2A.4) **Algebra and geometry.** The student connects algebraic and geometric representations of functions.	The student is expected to: (A) identify and sketch graphs of parent functions, including linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x)\ a^x$), and logarithmic ($f(x) = \log_a x$) functions, absolute value of x ($f(x) = \|x\|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = 1/x$); (B) extend parent functions with parameters such as a in $f(x) = a/x$ and describe the effects of the parameter changes on the graph of parent functions; and (C) describe and analyze the relationship between a function and its inverse.
Conic Sections	(2A.5) **Algebra and geometry.** The student knows the relationship between the geometric and algebraic descriptions of conic sections.	The student is expected to: (A) describe a conic section as the intersection of a plane and a cone; (B) sketch graphs of conic sections to relate simple parameter changes in the equation to corresponding changes in the graph; (C) identify symmetries from graphs of conic sections; (D) identify the conic section from a given equation; and (E) use the method of completing the square.
Quadratics: Representations	(2A.6) **Quadratic and square root functions.** The student understands that quadratic functions can be represented in different ways and translates among their various representations.	The student is expected to: (A) determine the reasonable domain and range values of quadratic functions, as well as interpret and determine the reasonableness of solutions to quadratic equations and inequalities; (B) relate representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions; and (C) determine a quadratic function from its roots or a graph.
Quadratics: Transformations	(2A.7) **Quadratic and square root functions.** The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations.	The student is expected to: (A) use characteristics of the quadratic parent function to sketch the related graphs and connect between the $y = ax^2 + bx + c$ and the $y = a(x - h)^2 + k$ symbolic representations of quadratic functions; and (B) use the parent function to investigate, describe, and predict the effects of changes in a, h, and k on the graphs of $y = a(x - h)^2 + k$ form of a function in applied and purely mathematical situations.

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Quadratics: Solving Equations	(2A.8) **Quadratic and square root functions.** The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.	The student is expected to: (A) analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems; (B) analyze and interpret the solutions of quadratic equations using discriminants and solve quadratic equations using the quadratic formula; (C) compare and translate between algebraic and graphical solutions of quadratic equations; and (D) solve quadratic equations and inequalities using graphs, tables, and algebraic methods.
Square Root Functions	(2A.9) **Quadratic and square root functions.** The student formulates equations and inequalities based on square root functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.	The student is expected to: (A) use the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describe limitations on the domains and ranges; (B) relate representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions; (C) determine the reasonable domain and range values of square root functions, as well as interpret and determine the reasonableness of solutions to square root equations and inequalities; (D) determine solutions of square root equations using graphs, tables, and algebraic methods; (E) determine solutions of square root inequalities using graphs and tables; (F) analyze situations modeled by square root functions, formulate equations or inequalities, select a method, and solve problems; and (G) connect inverses of square root functions with quadratic functions.

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Rational Functions	(2A.10) **Rational functions.** The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.	The student is expected to: (A) use quotients of polynomials to describe the graphs of rational functions, predict the effects of parameter changes, describe limitations on the domains and ranges, and examine asymptotic behavior; (B) analyze various representations of rational functions with respect to problem situations; (C) determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities; (D) determine the solutions of rational equations using graphs, tables, and algebraic methods; (E) determine solutions of rational inequalities using graphs and tables; (F) analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem; and (G) use functions to model and make predictions in problem situations involving direct and inverse variation.
Exponential and Logarithmic Functions	(2A.11) **Exponential and logarithmic functions.** The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.	The student is expected to: (A) develop the definition of logarithms by exploring and describing the relationship between exponential functions and their inverses; (B) use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior; (C) determine the reasonable domain and range values of exponential and logarithmic functions, as well as interpret and determine the reasonableness of solutions to exponential and logarithmic equations and inequalities; (D) determine solutions of exponential and logarithmic equations using graphs, tables, and algebraic methods; (E) determine solutions of exponential and logarithmic inequalities using graphs and tables; and (F) analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem.

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Geometry

(a) Basic understandings.

(1) Foundation concepts for high school mathematics. As presented in Grades K-8, the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics. Students continue to build on this foundation as they expand their understanding through other mathematical experiences.

(2) Geometric thinking and spatial reasoning. Spatial reasoning plays a critical role in geometry; geometric figures provide powerful ways to represent mathematical situations and to express generalizations about space and spatial relationships. Students use geometric thinking to understand mathematical concepts and the relationships among them.

(3) Geometric figures and their properties. Geometry consists of the study of geometric figures of zero, one, two, and three dimensions and the relationships among them. Students study properties and relationships having to do with size, shape, location, direction, and orientation of these figures.

(4) The relationship between geometry, other mathematics, and other disciplines. Geometry can be used to model and represent many mathematical and real-world situations. Students perceive the connection between geometry and the real and mathematical worlds and use geometric ideas, relationships, and properties to solve problems.

(5) Tools for geometric thinking. Techniques for working with spatial figures and their properties are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to solve meaningful problems by representing and transforming figures and analyzing relationships.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, connections within and outside mathematics, and reasoning (justification and proof). Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem solving contexts.

Basic Elements	(G.1) **Geometric structure.** The student understands the structure of, and relationships within, an axiomatic system.	The student is expected to: (A) develop an awareness of the structure of a mathematical system, connecting definitions, postulates, logical reasoning, and theorems; (B) recognize the historical development of geometric systems and know mathematics is developed for a variety of purposes; and (C) compare and contrast the structures and implications of Euclidean and non-Euclidean geometries.
Making Conjectures	(G.2) **Geometric structure.** The student analyzes geometric relationships in order to make and verify conjectures.	The student is expected to: (A) use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships; and (B) make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

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Axiomatic Systems	(G.3) **Geometric structure.** The student applies logical reasoning to justify and prove mathematical statements.	The student is expected to: (A) determine the validity of a conditional statement, its converse, inverse, and contrapositive; (B) construct and justify statements about geometric figures and their properties; (C) use logical reasoning to prove statements are true and find counter examples to disprove statements that are false; (D) use inductive reasoning to formulate a conjecture; and (E) use deductive reasoning to prove a statement.
Representations	(G.4) **Geometric structure.** The student uses a variety of representations to describe geometric relationships and solve problems.	The student is expected to select an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.
Patterns and Transformations	(G.5) **Geometric patterns.** The student uses a variety of representations to describe geometric relationships and solve problems.	The student is expected to: (A) use numeric and geometric patterns to develop algebraic expressions representing geometric properties; (B) use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles; (C) use properties of transformations and their compositions to make connections between mathematics and the real world, such as tessellations; and (D) identify and apply patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.
Solids: Representations	(G.6) **Dimensionality and the geometry of location.** The student analyzes the relationship between three-dimensional geometric figures and related two-dimensional representations and uses these representations to solve problems.	The student is expected to: (A) describe and draw the intersection of a given plane with various three-dimensional geometric figures; (B) use nets to represent and construct three-dimensional geometric figures; and (C) use orthographic and isometric views of three-dimensional geometric figures to represent and construct three-dimensional geometric figures and solve problems.
Coordinate Geometry	(G.7) **Dimensionality and the geometry of location.** The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.	The student is expected to: (A) use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures; (B) use slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons; and (C) derive and use formulas involving length, slope, and midpoint.

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Area, Surface Area, Volume	(G.8) **Congruence and the geometry of size.** The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations.	The student is expected to: (A) find areas of regular polygons, circles, and composite figures; (B) find areas of sectors and arc lengths of circles using proportional reasoning; (C) derive, extend, and use the Pythagorean Theorem; and (D) find surface areas and volumes of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations.
Properties of Planar and Solid Figures	(G.9) **Congruence and the geometry of size.** The student analyzes properties and describes relationships in geometric figures.	The student is expected to: (A) formulate and test conjectures about the properties of parallel and perpendicular lines based on explorations and concrete models; (B) formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models; (C) formulate and test conjectures about the properties and attributes of circles and the lines that intersect them based on explorations and concrete models; and (D) analyze the characteristics of polyhedra and other three-dimensional figures and their component parts based on explorations and concrete models.
Congruence	(G.10) **Congruence and the geometry of size.** The student applies the concept of congruence to justify properties of figures and solve problems.	The student is expected to: (A) use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane; and (B) justify and apply triangle congruence relationships.
Proportion and Similarity	(G.11) **Similarity and the geometry of shape.** The student applies the concepts of similarity to justify properties of figures and solve problems.	The student is expected to: (A) use and extend similarity properties and transformations to explore and justify conjectures about geometric figures; (B) use ratios to solve problems involving similar figures; (C) develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples using a variety of methods; and (D) describe the effect on perimeter, area, and volume when one or more dimensions of a figure are changed and apply this idea in solving problems.

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Presenter Preparation Checklist

Suggested Table set (per group of 4):

- ◻ Restickable notes of varying sizes
- ◻ Rulers
- ◻ Tape measures (metric)
- ◻ Highlighters
- ◻ Post-it flags
- ◻ Tape
- ◻ Masking tape
- ◻ Flip chart markers
- ◻ Pencils
- ◻ **Technology Tutorial** binder (one per computer)
- ◻ Chart paper

Manipulatives/Materials

Engage:

- ◻ Sticky dots
- ◻ Almanac—one per every two groups
- ◻ CBRs—one per every two groups
- ◻ Beach balls of different sizes—one set per every group of 4 participants at this station
- ◻ One-inch cubes—one per every two groups
- ◻ Meter stick—one per every two groups

Explore/Explain 1:

- ◻ Ramp—one per group
- ◻ Marble—one per group
- ◻ Cups or containers of varying sizes—one per group
- ◻ Measuring Tape (metric)—one per group
- ◻ Carbon paper or NCR form—one per group
- ◻ Hard flat plastic surface (for carpeted rooms only)—one per group
- ◻ Textbooks—two or three per group

Explore/Explain 3:

- ◻ CBL2—one per group
- ◻ Light probe—one per group
- ◻ Flashlight with fresh batteries—one per group
- ◻ Meter sticks—two or three per group
- ◻ Measuring Tape (metric, optional)—one per group (instead of meter sticks)
- ◻ Graph link cable
- ◻ Extra batteries for flashlight and CBL2

Elaborate:

- ◻ Sentence strips – blue and yellow, one of each per participant

Advanced Preparation

Engage:

- ☐ Copy with a color printer on cardstock and cut out Data Station Cards—A, B, C, D
- ☐ Cut out 36 one inch squares for Data Station D
- ☐ Chart Paper
  - Statements about technology with Likert scale—one per statement
  - Reflections on Data Venn Diagram—one per 12 participants

Explore/Explain 1:

- ☐ Build the ramp according to directions—one per group of participants

Explore/Explain 2:

- ☐ Copy the Geometer's Sketchpad® sketch **Golden Triangles.gsp** onto the desktop of each participants' computer.

Technology

- ☐ Presentation computer loaded with most recent update of:
  - PowerPoint (optional)
  - TI InterActive!
  - TI Connect
  - Excel
  - Word
  - Internet access
  - Hyperlink document (optional)
- ☐ Data projector
- ☐ Overhead projector
- ☐ One computer per two participants loaded with most recent update of:
  - Geometer's Sketchpad
  - TI InterActive!
  - TI Connect
  - Excel
  - Word
  - Internet access
  - Hyperlink document (optional)
- ☐ TI-83/84 overhead graphing calculator loaded with CBR/CBL program
- ☐ TI-83/84 calculator loaded with CBR/CBL program – one per participant
  - Graph link (optional)
  - DataMate program (optional)
- ☐ CBRs
- ☐ CBL2s – one per group of participants
  - Light Probe – one per group of participants
- ☐ Jumpdrives (optional)

Transparencies or PowerPoint Slides

Engage:

- □ Reflections on Data—Internet vs Almanac
- □ Reflections on Data—Calculator vs Hands-On
- □ Debriefing the Exploration of Data

Explore/Explain 1:

- □ Transparency 1: Ramp Setup
- □ Transparency 2: Collecting the Third Point

Elaborate:

- □ Teaching Strategies
- □ Transparency 1: Looks Like—Sounds Like
- □ Transparency 2: Looks Like, Sounds Like
- □ Student Research

Evaluate:

- □ Encouraging Judicious Use of Technology

Handouts

Prepare one folder for each participant to use through out the training. The handout for *Planning for Intentional Use of Data in the Classroom* from the Engage phase and the Explore/Explain phases should all be copied on the same particular color (i.e., green). The other handouts should be copied on different colors for each phase (i.e., light pink for the Engage, light blue for Explore/Explain 1, etc.). It also might be helpful to staple these colored pages together.

Engage:

- □ Data Station A Recording Sheet
- □ Data Station B Recording Sheet
- □ Data Station C Recording Sheet
- □ Data Station D Recording Sheet
- □ Reflections on Data—Internet vs. Almanac
- □ Reflections on Data—Calculator vs. Hands-on
- □ Debriefing the Exploration of Data
- □ Planning for Intentional Use of Data in the Classroom (copy on green paper)

Explore/Explain 1:

- □ Activity Pages: Flying Off the Handle
- □ Flying Off the Handle: Intentional Use of Data (copy on green paper)

Explore/Explain 2:

- □ Activity Pages: A Golden Idea
- □ A Golden Idea: Intentional Use of Data (copy on green paper)

Explore/Explain 3:

- □ Activity Pages: I've Seen the Light!

- □ I've Seen the Light!: Intentional Use of Data (copy on green paper)

Elaborate:

- □ Activity Page: The Doomsday Model

Evaluate:

- □ Gallery Walk Observations
  - Flying Off the Handle
  - A Golden Idea
  - I've Seen the Light!
  - The Doomsday Model

Leader Notes: Name Your Source!

Engage Phase

Purpose:
Provide participants the opportunity to investigate a variety of data sources. *Assess participants' experience and comfort with various avenues and tools for collecting data.* Compare and contrast technology-based data sources with technology-free data sources.

Descriptor:
Participants will rotate through four stations to gather data:

- Internet data sources
- Printed data sources, such as an almanac
- Calculator-based data collection tools
- Technology-free data collection tools

Upon completion of the activities at each station, participants will compare and contrast their experiences with Internet data sources and printed data sources. They will also compare and contrast their experiences with calculator-based data collection tools and technology-free data collection tools. Introduce participants to the formulation of questions that will spark data collection and investigation.

Duration:
1.5 hours

TEKS:
2A.1 (A) Identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations.

2A.1 (B) Collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

TAKS Objectives Supported by these Algebra 2 TEKS:
- Objective 1: Functional Relationships
- Objective 10: Mathematical Processes and Tools

Technology:
- Internet Website: *http://exploringdata.cqu.edu.au/datasets/oil_prod.xls*
- Calculator-based ranger (CBR) and graphing calculator

Materials:

Advanced Preparation:

- ✓ Create survey statements on chart paper for the recording of individual responses.
- ✓ Print Data Station Cards using a color printer.
- ✓ Recreate Venn diagrams from the Reflections on Data activity sheet on large chart paper.
- ✓ Create one set of Venn diagrams for every 12 participants.
- ✓ Cut out 36 one-inch squares for each Data Station D that will be made available to participants.
- ✓ Copy "Planning for Intentional Use of Data in the Classroom" activity sheets on green paper.

Presenter Materials: Internet access and projection device, overhead graphing calculator, PowerPoint slides or transparencies of transparencies and activity sheets

Per group: *Data Station A:* Computers with internet access
Data Station B: 1 *Time Almanac 2005* for each group of 4 participants that at this station.
Data Station C: CBR, graphing calculator, and different sized beach balls for each group of 4 participants at this station.
Data Station D: One-inch cubes and yard stick for each group of 4 participants at this station

Per participant: activity sheets

Leader Notes:

The goal of the Engage phase is to begin conversations about data. As teachers see the value of data and the mathematics that can be explored and reinforced through the use of data, they will begin to seek out data. Technology offers the tools to efficiently make sense of data. Technology also offers effective means for representing data so that analysis may take place. Encourage participants to interact with each other. The presenter(s) should move around the room facilitating the activity. Use the ***Facilitation Questions*** *to guide and redirect participants, as needed.*

Engage

1. *Record the following statements on chart paper. Post these statements around the room.*

> Technology offers the opportunity to strengthen mathematical learning in my classroom.
>
> Strongly Disagree ——————————————— Strongly Agree
>
> Students should learn first with paper-and-pencil methods and then with technology.
>
> Strongly Disagree ——————————————— Strongly Agree
>
> My students know how to discern which of these methods best serves the purposes of a given problem: mental strategies, paper-and-pencil techniques, and technology applications.
>
> Strongly Disagree ——————————————— Strongly Agree
>
> The best technology tool for the mathematics classroom is the graphing calculator.
>
> Strongly Disagree ——————————————— Strongly Agree

2. *As participants enter the session, direct them to respond to the posted statements by placing a marker, such as a sticky dot, in the location that best corresponds to their response. Use only one color of sticky dot for this activity.*

3. *As you provide a welcome and introduction to this professional development session, direct the participants' attention to the posted statements, sharing that continued reflection about these statements will be explored in greater detail during the course of this professional development.*

4. *Distribute a* ***Data Station Card*** *to each participant. Direct the participants to move to the station described on his or her card.*

> ***Archival data*** *are data that are not, under normal circumstances, subject to change. Examples of archival data include results from concluded research, medical records, and historical data.*
> ***Dynamic data*** *are data that are, under normal circumstances, subject to change. The data may be updated routinely or on request. An example of dynamic data is survey results that update based on each new response.*
> ***Categorical data*** *reflect data organized by category rather than by number. The frequencies of the categorical data are counted. Examples of categorical data include favorite color, voting, males/females, etc.*
> ***Numerical data*** *are data that reflect measurable, quantifiable attributes. The measures, rather than the attributes, form the data. Examples of numerical data include measures of length, measures of radio frequency, measures of time, etc.*

5. *After participants have moved to the appropriate station, model the activity at Station A for the whole group using a projection system so that participants understand the intent of the activity. Avoid walking the participants through the entire activity sheet so that the groups at Station A still have a meaningful learning experience. Demonstrate using* *http://exploringdata.cqu.edu.au/datasets/oil_prod.xls*.

Facilitation Questions

- What data are provided by this webpage? How would we record this information on the **Data Station A Recording Sheet**?
  Answers may vary.
- Are the data numerical, categorical, or both? How would we record this on the **Data Station A Recording Sheet**?
  Answers may vary.
- What relationships are described by this data? Why? How would we record this information on the **Data Station A Recording Sheet**?
  Answers may vary.

6. *Explain that the time allotted for each data station is 12 minutes. In these 12 minute segments, the participants should explore the given data source while recording observations and notes on the station's recording sheet. A count-down timer is a beneficial tool for keeping participants on task.*

7. *Walk to each data station, clarifying directions as necessary and prompting discussion as necessary.*

8. *After 12 minutes have passed, direct the participants to rotate to the next data station. Data Station D participants should move to Data Station A, Data Station A participants should move to Data Station B, etc. Allow approximately 3 minutes to transition between groups.*

9. *Repeat the rotation until each group as been at each data station. Continue to use the facilitation questions as needed.*

Facilitation Questions

Data Station A

- What numerical data have you found?
  The number of barrels of oil produced in given years.
- What relationships are found within the numerical data?
  Answers may vary.
- What trends do you notice?
  Answers may vary.
- How might you prompt students to represent the data?
  Answers may vary. Plot the data on a coordinate grid with or without technology.
- What questions might you pose to your students about the students' representations of the data?
  Answers may vary. Does the data appear to be linear? Can you draw a trend line? Can you find the line of best fit (regression line)?

Data Station B

- What numerical data have you found?
  Answers may vary. Lengths of general coastlines and tidal shorelines.
- What relationships are found within the numerical data?
  Answers may vary.
- What trends do you notice?
  Answers may vary.
- How might you prompt students to represent the data?
  Answers may vary. Plot the data on a coordinate grid with or without technology.
- What questions might you pose to your students about their representations of the data?
  Answers may vary. Does the data appear to be linear? Does the data appear to be non-linear?

Data Station C

- What numerical data did you generate?
  Answers may vary.
- What relationships are found within the numerical data? Why?
  Answers may vary.
- How might you summarize the data generated by your group?
  Answers may vary.
- How might you represent the data generated by your group?
  Answers may vary.
- How might you use these tools to generate two sets of data for comparison purposes?
  Answers may vary.
- To what real-life experiences might our students relate this data collection activity?
  Answers may vary.

Facilitation Questions

Data Station D

- What numerical data did you generate?
  Answers may vary.
- What categorical data did you generate?
  Answers may vary.
- What relationships are found within the numerical data? Why?
  Answers may vary.
- What relationships are found within the categorical data? Why?
  Answers may vary.
- How might you summarize the data generated by your group?
  Answers may vary.
- How might you represent the data generated by your group?
  Answers may vary.
- How might you use these tools to generate two sets of data for comparison purposes?
  Answers may vary.
- To what real-life experiences might our students relate this data collection activity?
  Answers may vary.

10. *Upon completing rotation through each station, reorganize participants into groups of 4. If using the* ***Data Station Cards,*** *regroup by color. Prompt the participants to complete the* ***Reflections on Data*** *activity sheet individually. Remind the participants that archival data are data that are preexisting in some form of document. Dynamic data are generated and updated as new data are collected. Allow approximately 5 minutes for the completion of these activity sheets.*

11. *While the participants are completing their individual* ***Reflections on Data*** *activity sheets, post 1 set of Venn Diagrams for every 12 participants.*

12. *Prompt participants to move to the chart paper Venn diagrams in groups of 12 by combining 3 existing groups of 4 participants. Tell participants that they will work silently in these groups of 12 to create summary Venn diagrams of the three groups' discussions.*

13. *Prompt the group to identify the person with the longest hair. This person will be the first recorder. Prompt this person to record one statement on the large chart paper Venn diagrams. The statement may be a personal observation or an observation from the group's Venn diagrams.*

14. *Prompt the participant to pass the marker to a new recorder, preferably a person who was not a member of his or her discussion group. This person will record a new statement on the Venn diagram. Prompt participants to continue this process until each participant has had an opportunity to record a statement. Participants may record new observations or statements that occur as a result of seeing the reflections of others.* ***Note****: Depending on time,*

you may choose to have multiple participants recording on the Venn diagrams at the same time.

Facilitation Questions

- Which similarities did each group note?
  Answers may vary.
- Which similarities were new to you?
  Answers may vary.
- Which differences did each group note?
  Answers may vary.
- Which differences were new to you?
  Answers may vary.
- What are the benefits of an archival data source?
  Answers may vary. The teacher is able to prepare models of representations to which students can compare their efforts.
- What are the benefits of a CBR data source over a technology-free data source?
  Answers may vary. The CBR provides dynamic data in a graphical representation.
- What are the benefits over a technology-free data source over a CBR data source?
  Answers may vary. Availability of technology doesn't determine what learning a teacher offers at what time.

15. *Distribute the **Debriefing the Exploration of Data** activity sheet. Prompt participants to reflect upon the discussions summarized by the Venn diagrams and record their responses to each of the questions posed on the activity sheet. After a few minutes of recording time, prompt the participants to share their responses with another participant. Debrief the responses in whole-group setting, keeping in mind that the goal of this phase of the professional development is to consider data.*

Facilitation Questions

- When might an internet data source support the learning of the math TEKS?
  Answers may vary.
- When might an almanac data source support the learning of the math TEKS?
  Answers may vary.
- Are trends more apparent in data resulting from an Internet or an almanac data source? Why?
  Answers may vary.
- What are the limitations of an Internet data source?
  Answers may vary.
- What are the limitations of an almanac data source?
  Answers may vary.
- How might these limitations impact the learning of the math TEKS?
  Answers may vary.
- What topics in Algebra 2 lend themselves to archival data?
  Answers may vary.

Facilitation Questions

- How do internet-based data sources serve to engage students in the learning process?
  Answers may vary.
- How might you use internet-based data sources to assess student learning?
  Answers may vary.
- Looking at the two Venn diagrams, how are the data sources related?
  Answers may vary.
- Looking at the two Venn diagrams, how are the data sources different?
  Answers may vary.

16. *Pose the questions listed below to the whole group. Explain to the participants that these questions serve as "filtering questions" when seeking to incorporate the use of data into classroom instruction.*
    a. *What TEKS in a particular unit of study are enhanced through the use of data?*
    b. *What data are required to enhance the study of these TEKS?*
    c. *What question(s) may be answered using this data?*
    d. *How does using data allow one to increase the rigor of the learning experience? How might using data move the learner from remembering, understanding, and applying to analyzing and evaluating?*
    e. *What type of data would be most useful for the stated TEKS?*
    f. *What setting will be available during instruction related to these mathematical goals?*
    g. *What actual data source(s) may prove helpful in enhancing mathematical learning related to these TEKS?*

17. *Distribute the* ***Planning for Intentional Use of Data in the Classroom*** *activity sheet to each participant. Share with the participants that these reflective questions form the basis for the* ***Planning for Intentional Use of Data in the Classroom*** *activity. Share with the participants that these filtering questions helped to develop each of the activities contained within this professional development. This template will serve as a reflection tool to summarize each activity that follows in order to identify elements that support the judicious use of technology.*

Data Station Cards

**Print in color.

Station A	Station B	Station C	Station D
Station A	Station B	Station C	Station D
Station A	Station B	Station C	Station D
Station A	Station B	Station C	Station D
Station A	Station B	Station C	Station D
Station A	Station B	Station C	Station D
Station A	Station B	Station C	Station D
Station A	Station B	Station C	Station D

Data Station A Recording Sheet

Data Source	http://exploringdata.cqu.edu.au/datasets/oil_prod.xls
How would you describe this set of data? Why?	
What relationships are found within this set of data? Why?	
How would you represent this data? Why?	
What question(s) can we pose to students that this set of data helps to answer?	
How might this data be used to extend what students already understand about our course content?	

Data Station B Recording Sheet

Data Source	*Time Almanac 2005*, "Coastline of the United States," page 502.
How would you describe this set of data? Why?	
What relationships are found within this set of data? Why?	
How would you represent this data? Why?	
What question(s) can we pose to students that this set of data helps to answer?	
How might this data be used to extend what students already understand about our course content?	

Data Station C Recording Sheet

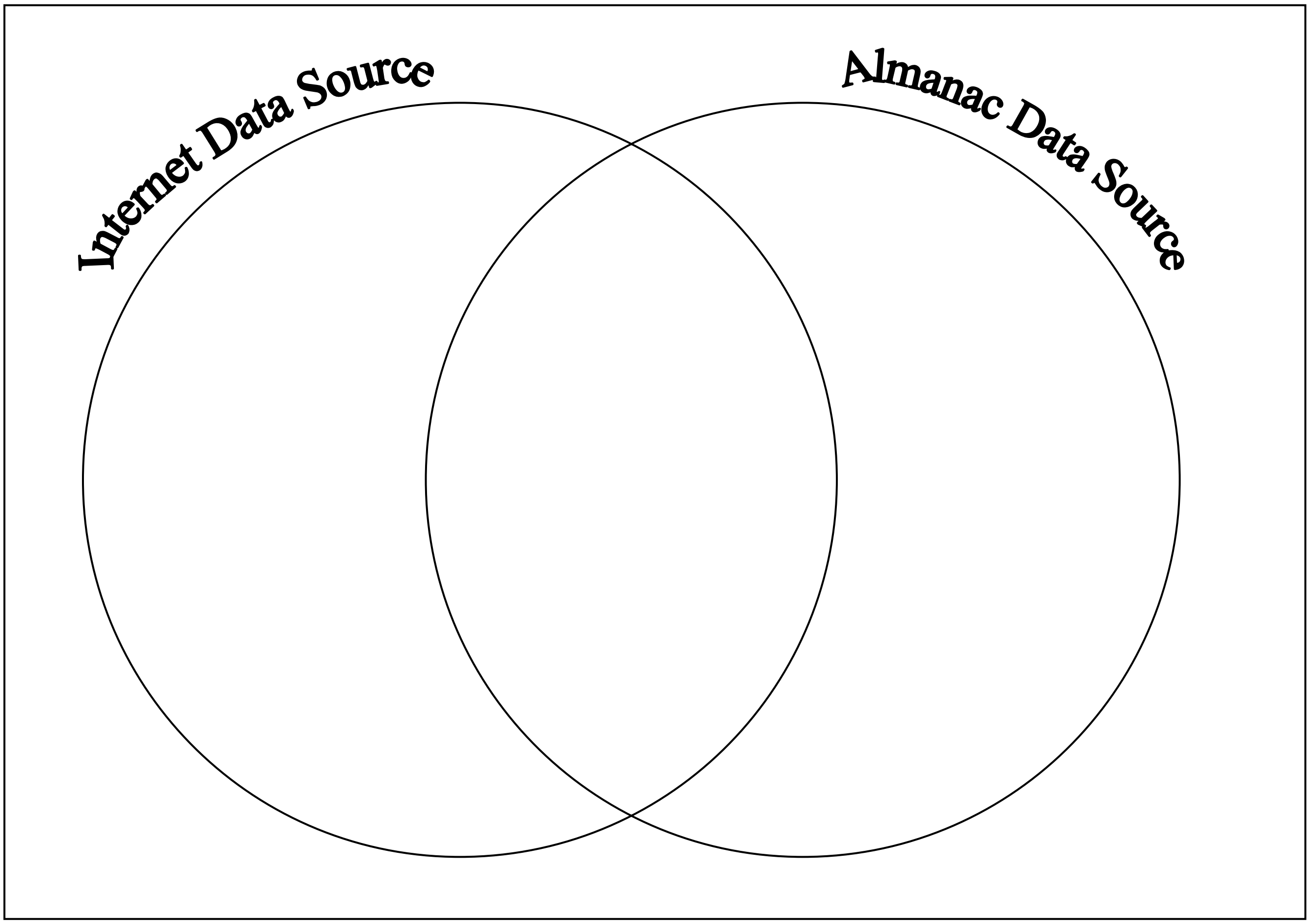
Data Source	CBR, graphing calculator, different sized beach balls
What set of data can you generate with these tools?	
What relationships are found within this set of data? Why?	
How would you represent this data? Why?	
What question(s) can we pose to students that this set of data helps to answer?	
How might this data be used to extend what students already understand about our course content?	

Data Station D Recording Sheet

Data Source	One-inch cubes, yard sticks
What set of data can you generate with these tools?	
What relationships are found within this set of data? Why?	
How would you represent this data? Why?	
What question(s) can we pose to students that this set of data helps to answer?	
How might this data be used to extend what students already understand about our course content?	

Reflections on Data

Complete the following Venn Diagram to compare and contrast the uses of the internet and an almanac as data sources.



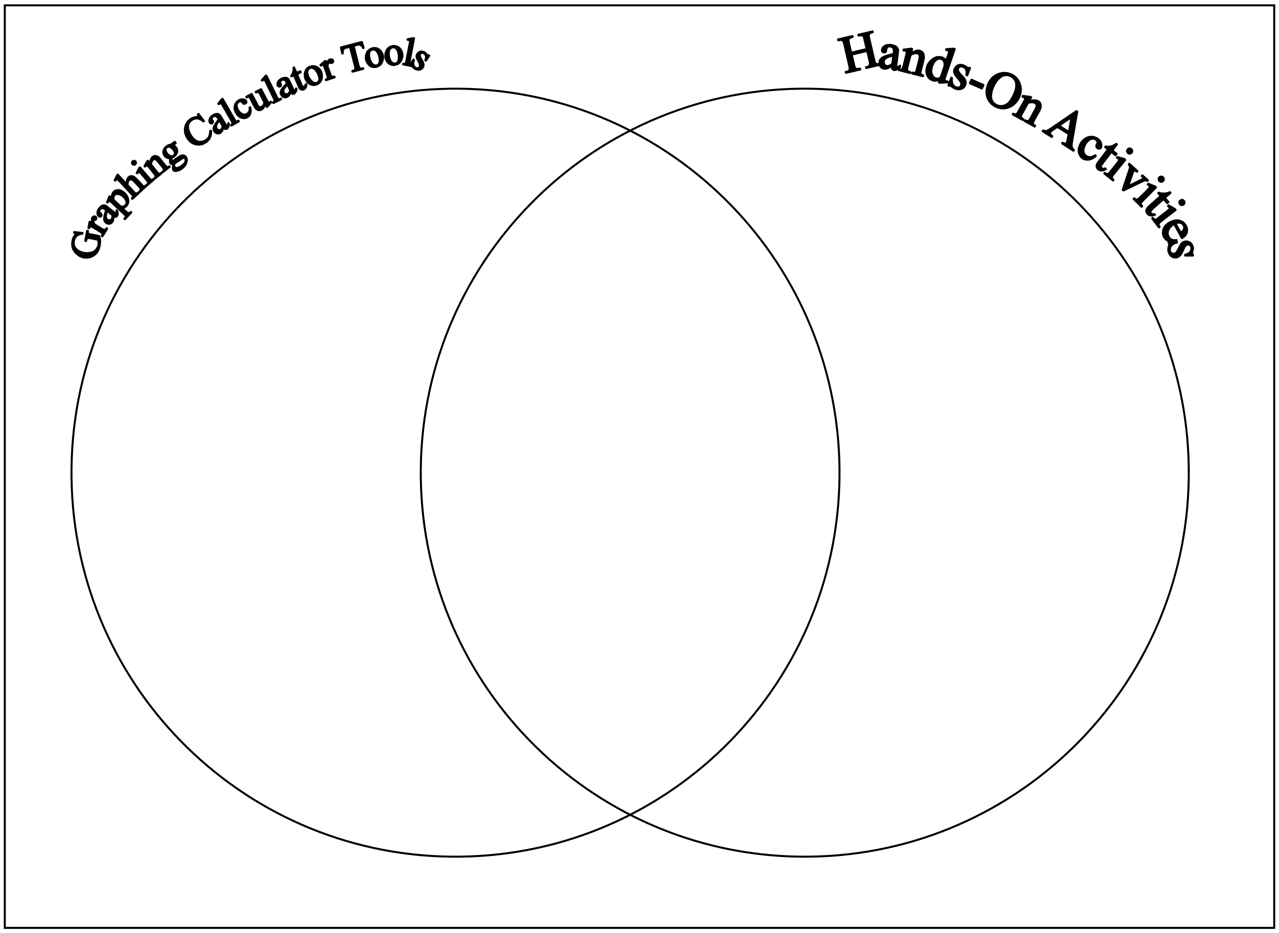
What are the benefits of using data found on the Internet?

What are the benefits of using data found in print sources such as an almanac?

How might teachers use these data sources in an Algebra 2 classroom?

Reflections on Data

Complete the following Venn Diagram to compare and contrast the uses of the graphing calculator tools and hands-on activities as data sources.



What are the benefits of using data resulting from graphing calculator tools?

What are the benefits of using data derived from hands-on activities?

How might teachers use these data sources in an Algebra 2 classroom?

Debriefing the Exploration of Data

1. What questions can we ask as reflective practitioners to determine the appropriateness of a data source for promoting mathematical learning?

2. How does the technology-based data offer an opportunity to strengthen mathematical learning?

3. How might hands-on activities complement the judicious use of technology?

4. What paper-and-pencil methods do students need to know to make sense of the data we explored?

Planning for Intentional Use of Data in the Classroom

TEKS			
Question(s) to Pose to Students	**Math**		
	Tech		
Cognitive Rigor		Knowledge	
		Understanding	
		Application	
		Analysis	
		Evaluation	
		Creation	
Data Source(s)		Real-Time	
		Archival	
		Categorical	
		Numerical	
Setting		Computer Lab	
		Mini-Lab	
		One Computer	
		Graphing Calculator	
		Measurement-Based Data Collection	
Bridge to the Classroom			

Leader Notes: Flying Off the Handle

Explore/Explain Cycle I

Purpose:

Investigate generating and solving systems of equations. Use graphing calculator technology to generate a quadratic function by solving a system of equations. Apply this quadratic function to solve a problem.

Descriptor:

When a marble rolls down a ramp then off the edge, it will exhibit projectile motion until it reaches the ground. Participants will overlay a coordinate system to this problem situation. By finding three data points (coordinates of the point where the marble leaves the ramp, coordinates of the point where the marble hits the ground, and coordinates of the point where the marble hits a chair or desk placed in its path), participants will generate a quadratic function. They will use this quadratic function to predict where they need to place a cup so that the marble will land inside the cup.

Duration:

2 hours

TEKS:

a(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

a(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

2A.1(B) Collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

2A.3(A) Analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems.

2A.3 (B) Use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.

2A.3 (C) Interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.

2A.4(A) Identify and sketch graphs of parent functions, including linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$) functions, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = \frac{1}{x}$).

2A.4(B) Extend parent functions with parameters such as a in $f(x) = \frac{a}{x}$ and describe the effects of the parameter changes on the graph of parent functions.

2A.6(B) Relate representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions.

2A.6(C) Determine a quadratic function from its roots or a graph.

2A.8(A) Analyze situations involving quadratic functions and formulate quadratic equations or inequalities to solve problems.

2A.8(C) Compare and translate between algebraic and graphical solutions of quadratic equations.

2A.8(D) Solve quadratic equations and inequalities using graphs, tables, and algebraic methods.

TAKS Objectives Addressed by these Algebra 2 TEKS:

- Objective 1: Functional Relationships
- Objective 2: Properties and Attributes of Functions
- Objective 10: Mathematical Processes and Mathematical Tools

Technology:

- Graphing Calculator

Materials:

Advanced Preparation: The ramp should be constructed before the presentation (see directions following); Transparencies 1 and 2

Presenter Materials: projector (computer or overhead) for graphing calculator

Per group: ramp, marble, tape measure, desk or chair, carbon paper (or NCR form), one sheet of copy paper, hard flat plastic surface (for carpeted rooms only), tape, one cup, 2 or 3 textbooks

Per participant: graphing calculator, activity sheets

Leader Notes:
Graphing calculators should be a part of the high school mathematics classroom culture in Texas since the Texas Essential Knowledge and Skills for every high school mathematics course

require students to "use technology…to model mathematical situations to solve meaningful problems." Furthermore, testing regulations for the Texas Assessment of Knowledge and Skills require students to have access to graphing calculators during the test. In this lesson, participants will collect data manually then use a graphing calculator to solve a meaningful problem.

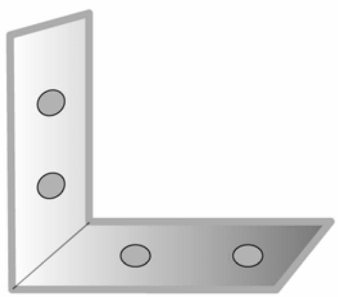
Marble Ramp Construction

1 - piece of 2 x 2 x 8′ wood (See Figure 1)

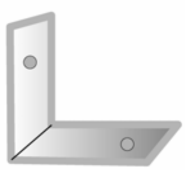
1 - 8′ piece of corner molding (will actually use ~5′)

2 - straight 3″ connectors

4 - 90° 3″ brackets to use as legs



1 - 90° bracket ($1\frac{1}{2}$" x $1\frac{1}{2}$")



18 - wood screws ($\frac{1}{2}$" - #10)

4 - wood screws (2″ - #10)

10 - small brads ($\frac{1}{2}$")

wood filler (optional)

hammer

Phillips head screwdriver

saw

wood glue

miter box

Instructions:

1. Gather the materials shown above.

2. Cut the 2 x 2 piece of wood into three sections cutting 45° angles as shown.

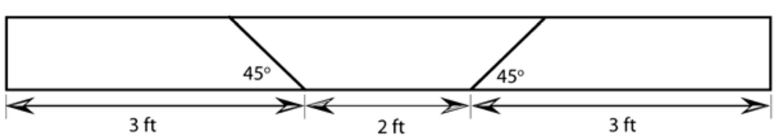
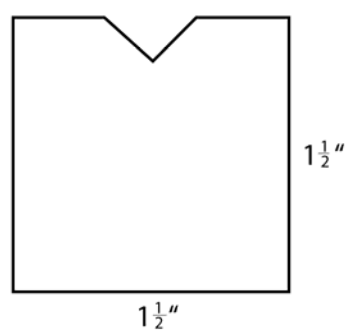


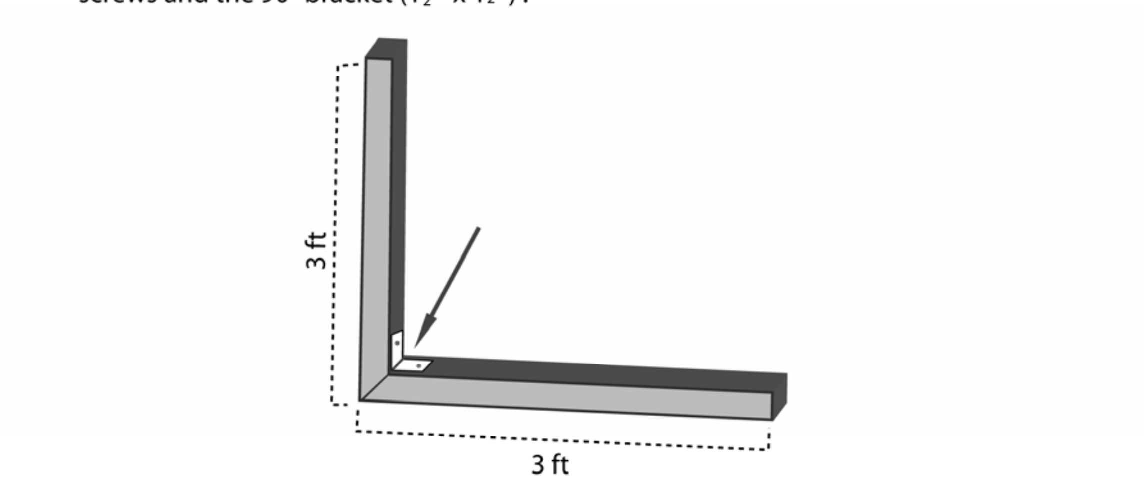
Figure 1

3. Make a groove in the 2 x 2 to hold the molding. (optional)

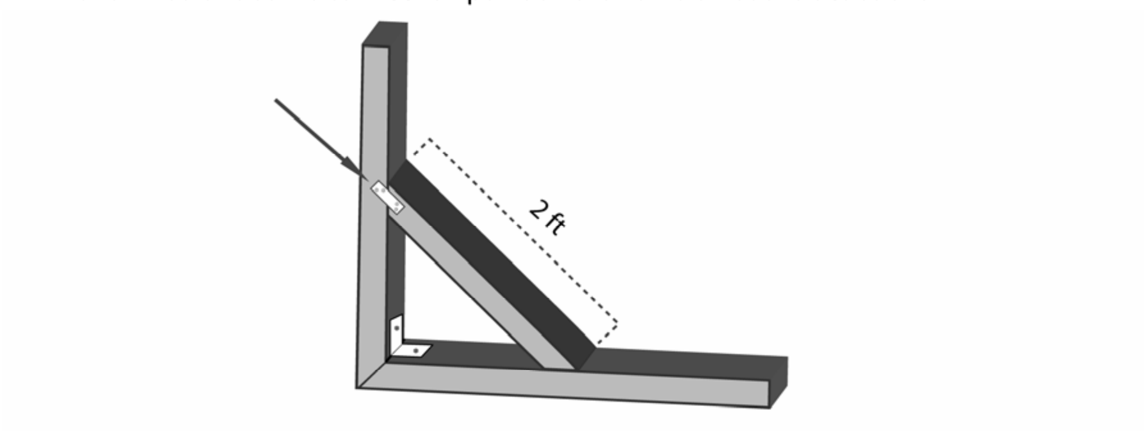


Note: Actual dimensions of 2 x 2 with optional notch

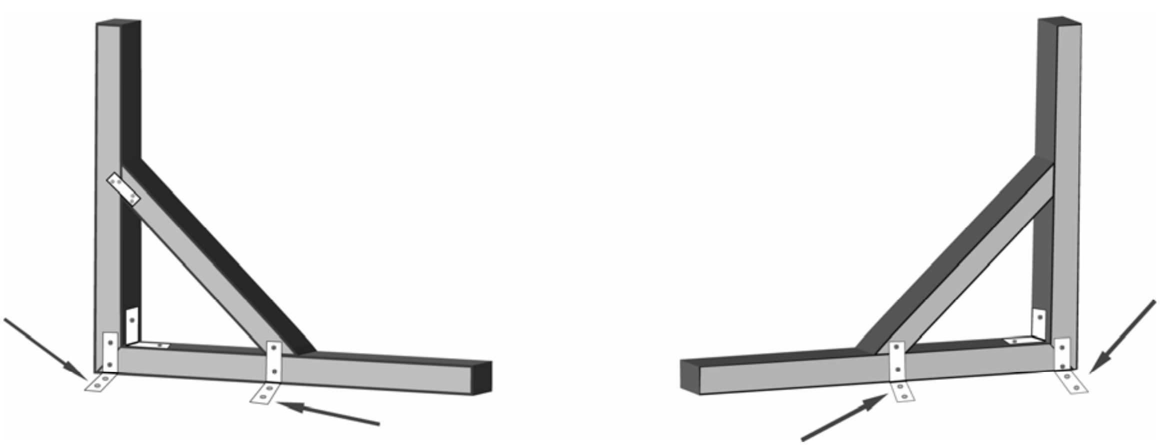
4. Join the two 3 ft pieces of wood together to form an L-shaped frame using the $\frac{1}{2}$" screws and the 90° bracket ($1\frac{1}{2}$" x $1\frac{1}{2}$").



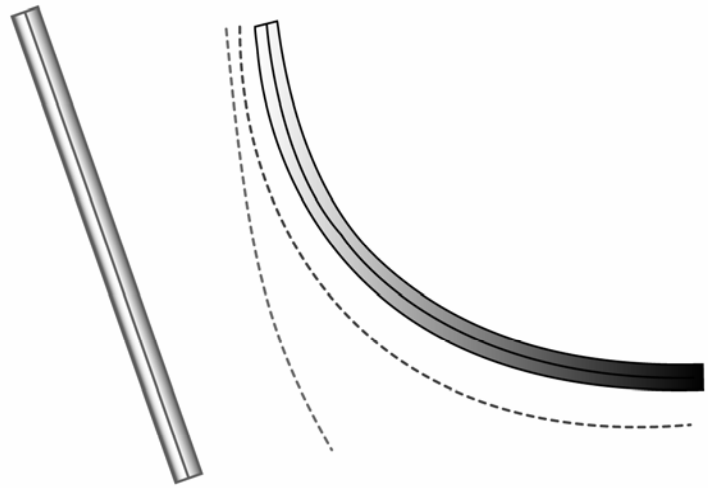
5. Attach the 2 ft piece of wood to the L-shaped frame with the 3" straight connectors and $\frac{1}{2}$" screws at the connection point on the frame on both sides as shown.



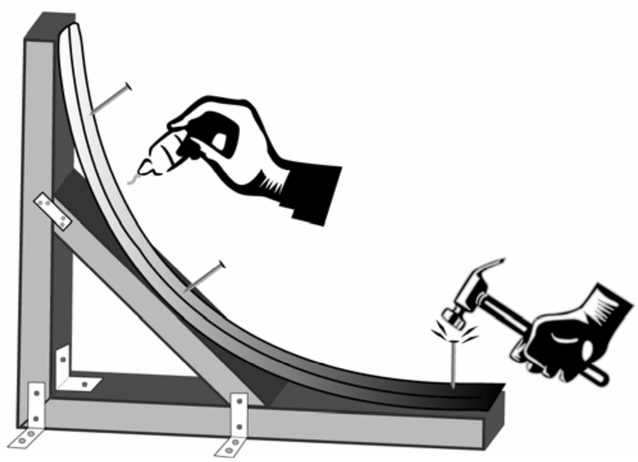
6. Attach the four 90° 3" brackets (legs) at the four positions on the frame with $\frac{1}{2}$" screws.



7. Bend the flexible corner molding so that it is in a curved shape.

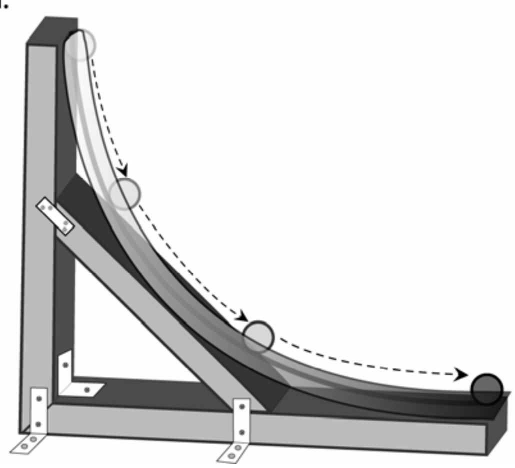


8. Glue and nail the molding to the frame using a nail set.



9. Fill in any nail holes with wood filler and sand as necessary. *(optional)*

10. The finished product will resemble the figure below and the marble will move as illustrated.



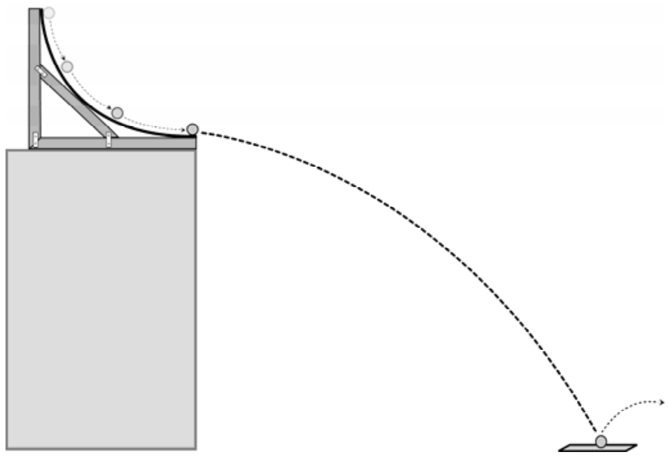
Explore

Posing the Problem:

In Hollywood movies, a classic way to end a car chase is to have a car drive off a cliff or into a canyon, with the hero jumping out of the car at the last minute to safety. Movie directors need to know exactly where the car will land so that they can have cameras in place to capture the motion on film. They also need to know where the car will be as it falls from the edge of the cliff to the floor of the canyon below so that they can have cameras in place there, too. How can we model this motion? How can we harness technology to apply this model to pinpoint the specific location of a moving object at any time?

To answer these questions, let's build a model that will enable us to simulate a car driving off a cliff. Use a wooden ramp and marble to simulate the car's motion. How would we develop a function rule to determine the placement of the cup on top of a stack of textbooks?

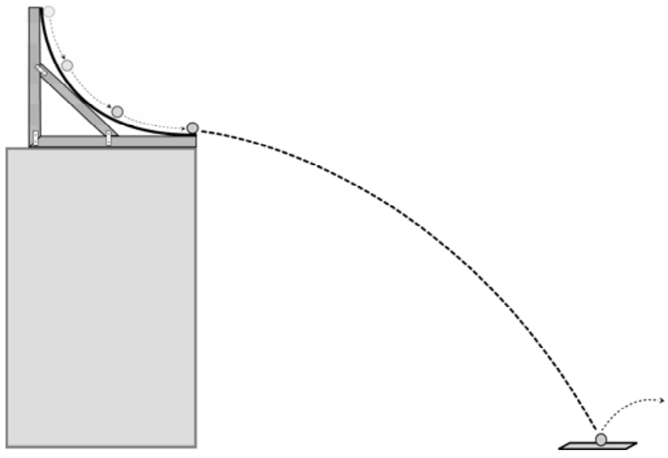
Note to Leader: when setting up the exploration, be sure to stress to participants that, as with producing movies, they will only get one shot to land the marble inside the cup. Hence, it is important that they collect data and build an accurate model.



Obtaining and Analyzing the Data:

Note to Leader: Set up the ramp on a high table. Roll the marble down the ramp and let it hit the floor to demonstrate the motion of the marble.

Display Transparency 1: Ramp Setup.



1. **Let the floor represent the *x*-axis and the end of the ramp be contained on the *y*-axis. Where would the origin of this coordinate system be?**
   The origin is the point on the floor directly beneath the end of the ramp.

2. **In this coordinate system, what do *x* and *y* represent?**
   x represents the horizontal distance from the end of the ramp and y represents the height above the floor.

Facilitation Questions
- Is there a dependency relationship between the *x* and *y* variables?
  No. The variables are related, but there is not a clear dependency between x and y.
- What is the relationship between the *x* and *y* variables?
  Both x and y represent distances that are dependent on the time that has elapsed since the marble left the end of the ramp.

3. **Consider the path of the marble. Based on your coordinate system, what does the *y*-intercept represent? What are the coordinates of the *y*-intercept? Record the coordinates as a point in the table.**
   The y-intercept is the point at the end of the ramp. Its x-coordinate is 0 and its y-coordinate is the height of the end of the ramp above the floor. The coordinates are recorded in the table shown with Question 6.

Facilitation Question
- Does the precision of measurement matter?
  Yes. The greater the precision of measurement of the distances, the greater the accuracy of the model.

4. **Based on your coordinate system, what does the *x*-intercept represent?**
   The x-intercept is the point where the marble lands on the floor.

Roll the marble down the ramp and notice where it strikes the floor. Tape a piece of carbon paper on top of a piece of typing paper (carbon side down) where the marble touched the floor. Tape the paper to the floor.

Note to Leader: *If you are working in a room with carpeted floors, you will need to have a hard plastic or wooden surface to put under the paper. Otherwise, the impact of the marble will be dampened by the carpet and the marble will not leave a mark on the paper.*

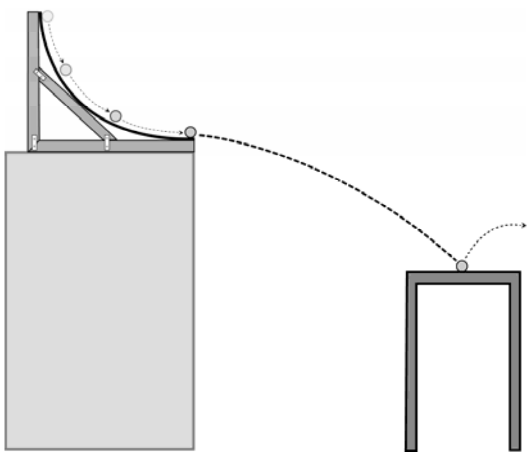
Roll the marble down the ramp and let it strike the paper on the floor. Repeat at least twice so that you have data for at least 3 trials.

Facilitation Questions

- Does the marble always land in the same spot? Why or why not?
  Typically, no. Many factors can contribute to the motion of the marble, including friction with the ramp, the wobbling motion of the marble while falling down the ramp, or the curvature of the ramp itself.
- How can you determine the "average" location where the marble hits the floor?
  Answers may vary. Participants should find a middle value that represents where the marble ought to reach the floor, allowing for some variance in location. Some participants may measure the three distances then find the arithmetic mean.

5. What are the coordinates of the *x*-intercept? Record the coordinates as a point in the table.

6. Place a chair or desk between the ramp and the point of impact on the floor. Repeat your data collection procedure to find the *x*- and *y*-coordinates of the point of impact on the chair or desk. Record your third data point in the table.
Display Transparency 2: Collecting the Third Data Point.



Sample Answers (in inches):

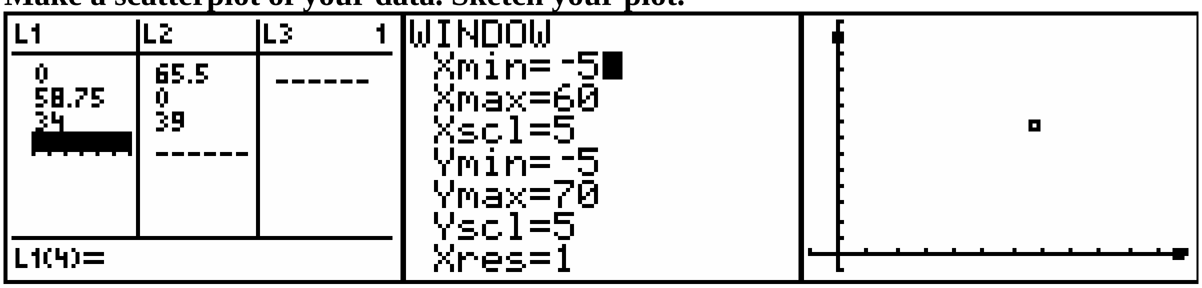
Horizontal Distance (x)	Height of the Marble (y)
0	*65.5*
58.75	*0*
34	*39*

Facilitation Questions

- Where will the marble land on the chair or desk?
  Roll the marble down the ramp to observe where the marble lands.
- Where should you place the carbon paper?
  Place the carbon paper where the marble lands on the chair or desk.
- In your coordinate system, what would the *x*-value represent?
  The x-value is the horizontal distance from the end of the ramp to the point where the marble lands on the chair or desk.
- In your coordinate system, what would the y-value represent?
  The y-value is the vertical distance between the point where the marble lands on the chair or desk and the floor.

7. **What kind of functional relationship do you think exists between the horizontal distance and the vertical distance of the marble?**
   Participants should predict a quadratic relationship due to the nature of free-fall and projectile motion.

8. **Make a scatterplot of your data. Sketch your plot.**



9. **Use the coordinates of the three data points to write a function rule that could be used to predict the height of the marble, *y*, when it is a horizontal distance, *x*, from the ramp. Explain how you found your function.**
   The quadratic function modeling the sample data is $y = -0.0136x^2 - 0.3185x + 65.5$.
   Methods of finding the function will vary. Participants could write a system of equations in standard form then use matrices to solve the system. Participants could also use transformations on the parent function $y = x^2$ *in order to fit a curve to the data. Quadratic regression could also be used to find a function rule, depending on the nature of the course.*

Facilitation Questions

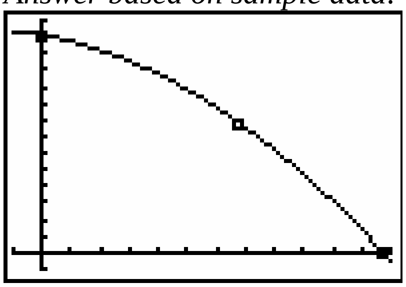
- Based on your answer to Question 7, what is the standard form for that type of function?
  The standard form for a quadratic equation is $ax^2 + bx + c = 0$.
- How could you set up a system of equations to solve for the parameters of your standard form equation?
  Substitute the known x- and y-values for each ordered pair and simplify.
- How could you solve this system of equations?
  Answers may vary. Since the numbers are rather unwieldy, matrices could be a good tool to solve this system of equations.

Facilitation Questions, continued

- Does the graph of the parent function model the data well?
  No.
- What transformations could you do to the parent function to obtain a model that fits the data well?
  A vertical reflection across the x-axis, a vertical shift of b units (b represents the y-coordinate of the y-intercept), and a vertical compression
- How could you use technology to make solving the problem easier?
  A graphing calculator can do matrix operations. Also, the graphing calculator and Excel will use quadratic regression features to generate a quadratic model for the data.

10. Graph your function rule over your scatterplot and sketch your graph. Is the function rule a good fit? How can you tell? If not, how can you revise your function rule so that it is a better fit?

Answer based on sample data:



Facilitation Question

- What transformations could you do to your model to obtain a model that better fits the data?
  A vertical reflection across the x-axis, a vertical shift of b units (b represents the y-coordinate of the y-intercept), and a vertical compression.

11. Place your cup on top of three textbooks. Where do you need to place the cup so that the marble will roll off the ramp and land inside the cup? Justify your choice.

Responses will vary depending on the size of the cup and textbooks. Participants should stack the textbooks and place the cup on top then measure the height of the cup above the ground. Using this height, they should determine the horizontal distance from the end of the ramp to the center of the top of the cup.

Facilitation Questions

- What can you directly measure regarding the cup and textbooks?
  The height of the cup and thickness of the textbooks can be measured.
- Where will the marble travel to land inside the cup?
  The marble must clear the front lip of the cup, so aiming for the center of the cup will get the marble inside the cup.
- What are the coordinates of this part of the cup? How do you obtain them?
  The coordinates are obtained by substituting the known y-value into the function rule and solving for x. This value will be the horizontal distance that the center of the cup will need to be from the edge of the ramp.

12. Test your prediction. Was your prediction correct? Why or why not? If not, revise your prediction and test it again.
Note to Leader: *Weigh the cup down then pad the bottom of the cup with tissue or crumpled up napkins so that the marble lands inside the cup without bouncing out or knocking the cup over.*

Facilitation Questions

- Should you move your cup toward or away from the ramp? Why?
Answers may vary. If the marble lands in front of the cup, the cup will need to be moved closer. If the marble lands behind the cup, the cup will need to be moved farther.
- Is your function model correct? Why or why not?
Answers may vary. The accuracy and precision of measurement of the original three points will greatly impact the accuracy of the function model.
- How can we generate a better function model?
Perform transformations on the model to yield a better fit. If this does not work, participants may need to recollect their three data points, paying attention to accuracy and precision of measurement, and generate a new function model based on their new data.

Explain

In this phase, use the debrief questions to prompt participant groups to share their responses to the data analysis. At this stage in the professional development, participants should be familiar with using the graphing calculator. Some participants may be familiar with using a spreadsheet such as Excel to analyze data.

1. How did you develop your function rule? Why did you choose this method?
Ask participants to discuss their methods and their reasons for making that choice. If none of the participants choose a particular method, ask participants why no one made that choice.

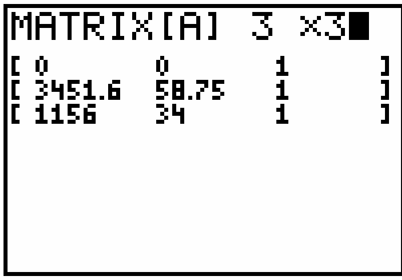
Using Matrices:
Beginning with the polynomial form of a quadratic function, $y = ax^2 + bx + c$*, substitute values of x and y for the data points:*

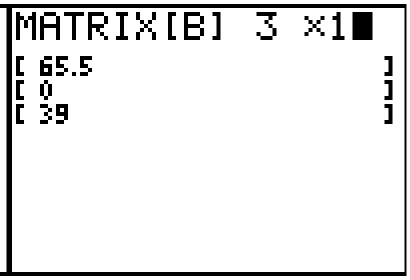
$$65.5 = a(0)^2 + b(0) + c \qquad 65.5 = c$$
$$0 = a(58.75)^2 + b(58.75) + c \qquad 0 = 3451.5625a + 58.75b + c$$
$$39 = a(34)^2 + b(34) + c \qquad 39 = 1156a + 34b + c$$

Write a matrix equation to represent this system.

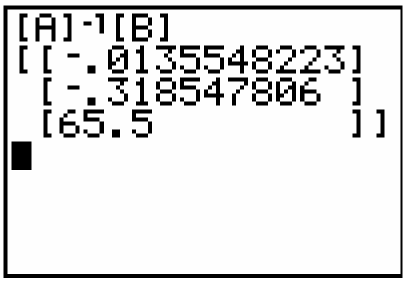
$$\begin{bmatrix} 0 & 0 & 1 \\ 3451.5625 & 58.75 & 1 \\ 1156 & 34 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 65.5 \\ 0 \\ 39 \end{bmatrix}$$

Enter the coefficient matrix into the calculator's matrix A and the column matrix of the known terms into matrix B. For detailed instructions, see "Technology Tutorial: Entering Data into Matrices."



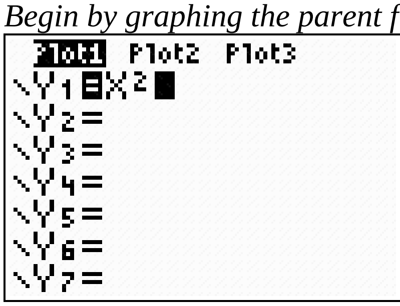


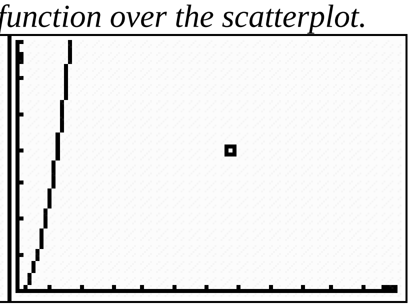
To solve the system, we need to left-multiply the matrix equation by the inverse of the coefficient matrix, or $[A]^{-1}$*:*



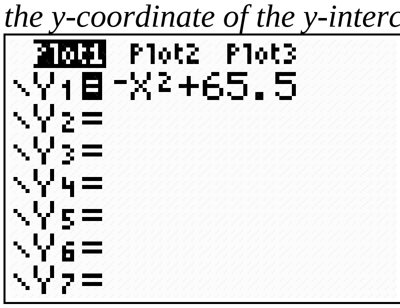
Thus, the quadratic function modeling our data is $y = -0.0136x^2 - 0.3185x + 65.5$.

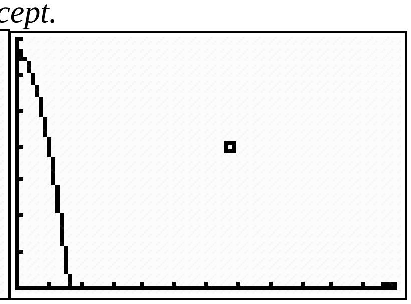
Using Transformations:
Begin by graphing the parent function over the scatterplot.



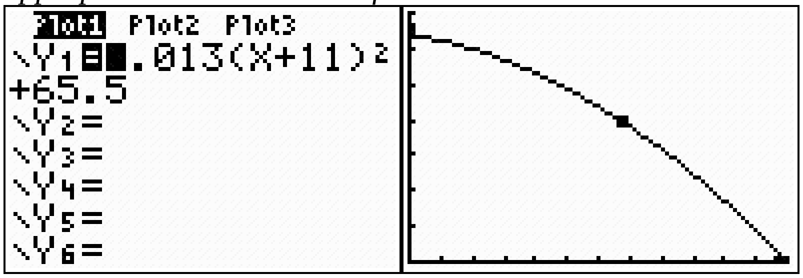


Reflect the parent function over the x-axis then vertically shift the parabola by the value of the y-coordinate of the y-intercept.

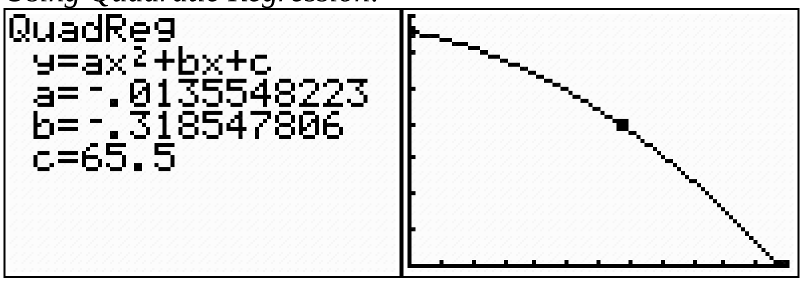




Continue using transformations, including vertical stretches or compressions, until an appropriate model has been found.

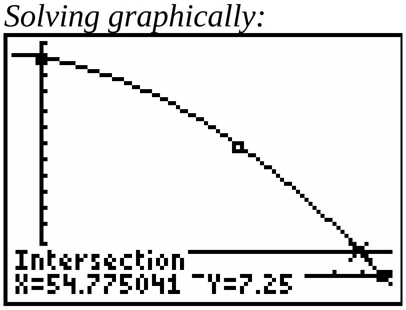


Using Quadratic Regression:

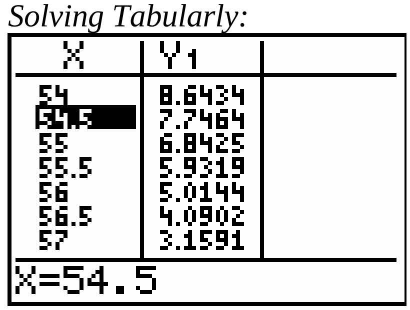


2. **How did you determine the location for your cup? Why did you choose this method?**
Responses will vary depending on the height of the cup and the thickness of the textbooks. In this example, assume that the textbooks are 1 inch thick each and that the cup is 4.25 inches tall. This assumption gives a total height of 1 + 1 + 1 + 4.25 = 7.25 inches. Thus, we need to find the horizontal distance when the marble is 7.25 inches above the ground; i.e., we need to solve for x when y = 7.25.

Solving graphically:



Solving Tabularly:



The cup needs to be placed so that the center of the cup is about $54\frac{3}{4}$ *inches from the foot of the ramp.*

3. **How accurate were your predictions? If they needed revision, how did you decide on the revisions?**
Responses will vary. If a cup has a small opening and participants are not careful of their precision of measurement, they may need to re-measure their data points to get a more accurate model. Some participants may report that they incorrectly solved the equation and had to solve the equation using another method.

4. **What problem-solving strategies did you use to solve this problem?**
   Answers may vary. Possible answers may include "solving a simpler problem," "using a model," or "writing an equation."

5. **Could you use a technology other than the graphing calculator to solve this problem?**
   Answers may vary. See "Technology Tutorial: Flying Off the Handle" for details.

Note to Leader: *Record or have a participant volunteer record the responses to Questions 6 and 7 on chart paper to use in the Elaborate phase of the professional development.*

6. **What are the relative advantages and disadvantages of using a graphing calculator to solve this problem?**
   Responses may vary.

   The matrix operations on the calculator make it easy to solve a matrix equation. The graphical analysis features of the calculator make it easy to use the graph to solve problems by tracing and calculating the intersection of lines.

   However, the small screen is difficult to see, and the axes in the window cannot be labeled.

7. **What are the relative advantages and disadvantages of using a spreadsheet to solve this problem?**
   Responses may vary.

   The regression equation is calculated quickly on the spreadsheet. The axes can be clearly labeled with numbers and text labels. Labeled axes help the participant to use the graph to estimate solutions to problems that can be solved graphically. The graph can be enlarged or reduced then copied and pasted into other computer documents such as a Word or PowerPoint document to communicate the solution to a problem.

   However, the participant is limited to the regression equations available in the spreadsheet. There are no graphical analysis features in most spreadsheets, so only estimates rather than exact solutions can be obtained graphically.

8. **What TEKS does this activity address?**
   Participants should brainstorm a list of TEKS that they believe they have covered in this activity. The Leader Notes contain a comprehensive list of the TEKS addressed in this phase of the professional development. If participants do not mention some of these TEKS, then ask them how the activity also covers them.

9. **How does the technology that you used enhance the teaching of those TEKS?**
   Responses may vary. However, participants should note that using technology enables them to explore a mathematical concept to a much deeper level. In this case, using technology made solving the problem significantly easier than parallel methods using pencil and paper. Technology makes rich mathematics accessible to a variety of learning styles.

Flying Off the Handle: Intentional Use of Data

1. At the close of *Flying Off the Handle*, distribute the **Intentional Use of Data** activity sheet to each participant.

2. Prompt the participants to work in pairs to identify those TEKS that received greatest emphasis during this activity. Also prompt the participants to identify two key questions that are emphasized during this activity. Allow four minutes for discussion.

Facilitation Questions
- Which mathematics TEKS form the primary focus of this activity?
- What additional mathematics TEKS support the primary TEKS?
- How do these TEKS translate into guiding questions to facilitate student exploration of the content?
- How do your questions reflect the depth and complexity of the TEKS?
- How do your questions support the use of technology?

3. As a whole group, discuss responses for two to three minutes.

4. As a whole group, identify the level(s) of rigor (based on Bloom's taxonomy) addressed, the types of data, the setting, and the data sources used during this Explore/Explain cycle. Allow three minutes for discussion.

Facilitation Question
- What attributes of the activity support the level of rigor that you identified?

5. As a whole group, discuss how this activity might be implemented in other settings. Allow five minutes for discussion.

6. Prompt the participants to set aside the completed Intentional Use of Data activity sheet for later discussion. These completed activity sheets will be used during the Elaborate phase as prompts for generating attributes of judicious users of technology.

Facilitation Questions
- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) per participant?
- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) per small group of participants?
- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) for the entire group of participants?

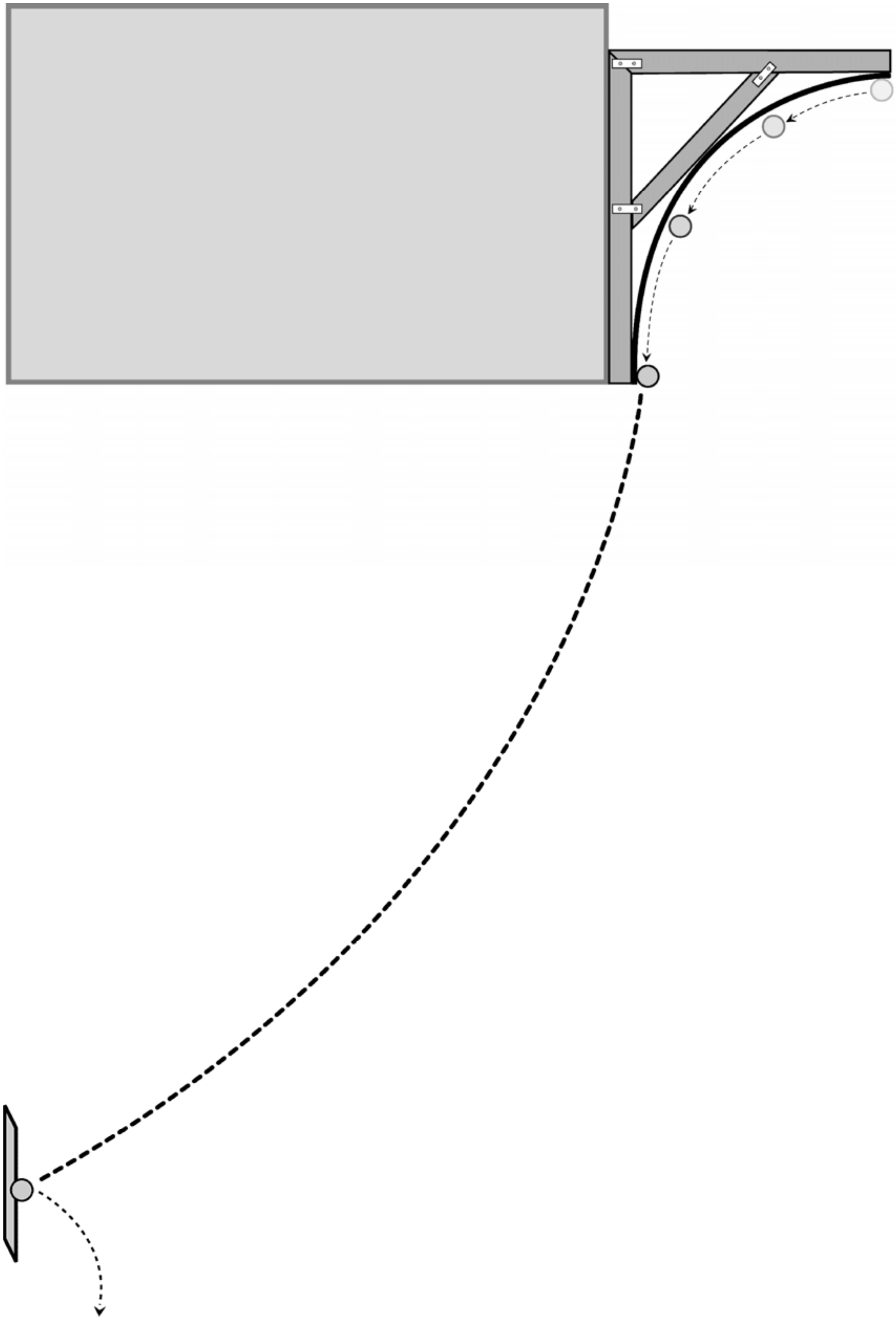
Facilitation Questions

- How would this activity change if we had used computer-based applications instead of graphing calculators?
- How might we have made additional use of available technologies during this activity?
- How does technology enhance learning?

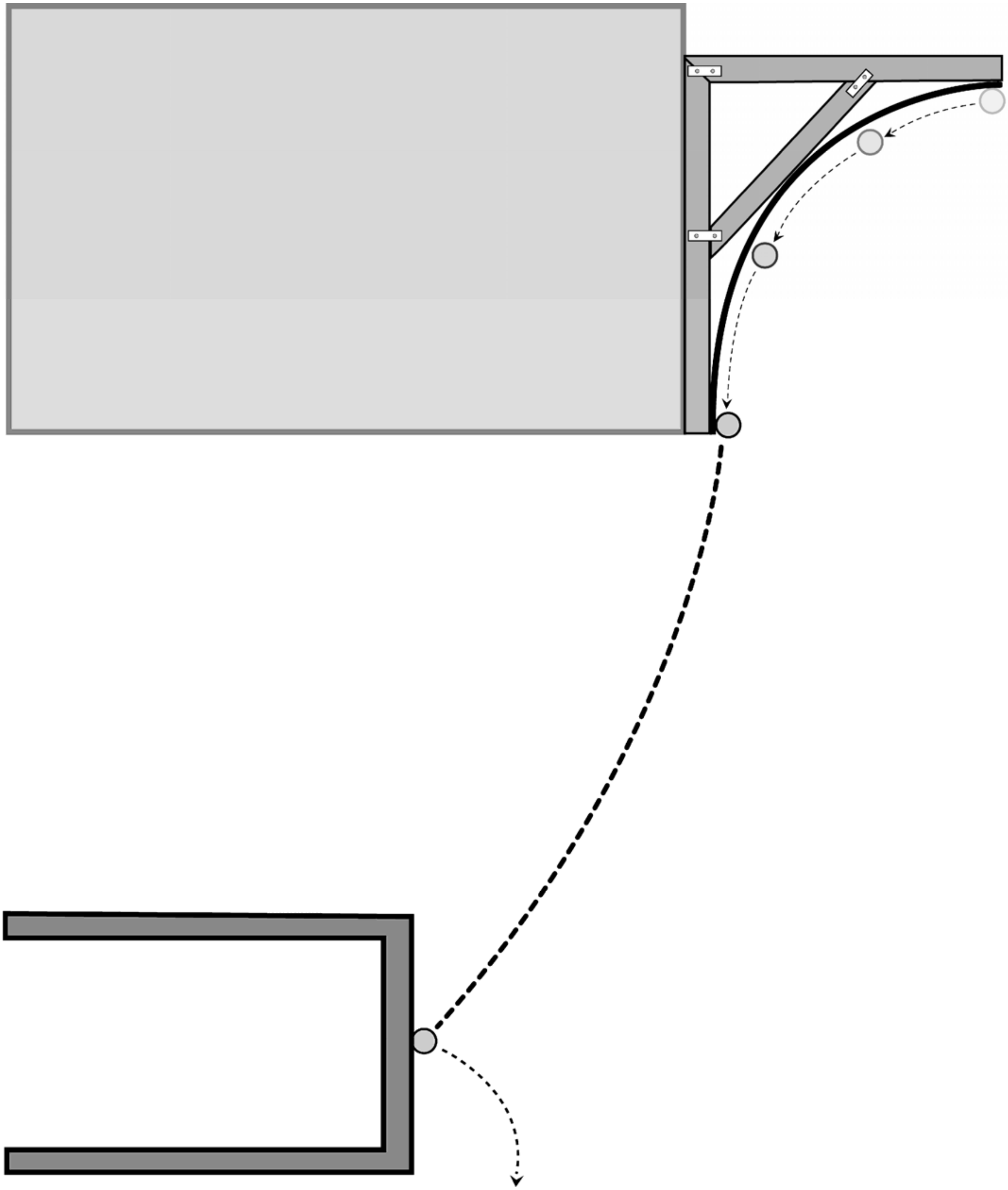
Sample Responses:

TEKS		*a(5), a(6), 2A.1B, 2A.3A, 2A.3B, 2A.3C, 2A.4A, 2A.4B, 2A.6B, 2A.6C, 2A.8A, 2A.8C, 2A.8D*	
Question(s) to Pose to Students	**Math**	*What type of relationship models the data you collected?* *How can you transform the parent function in order to better fit the data points?*	
	Tech	*How did technology help you with the analysis of data?* *How did technology help you to solve the problem?*	
Cognitive Rigor		Knowledge	√
		Understanding	√
		Application	√
		Analysis	√
		Evaluation	√
		Creation	√
Data Source(s)		Real-Time	
		Archival	
		Categorical	
		Numerical	*Three data points collected by measurement*
Setting		Computer Lab	
		Mini-Lab	
		One Computer	
		Graphing Calculator	*Used to analyze the data, either via transformations, matrices to solve a system of equations, or quadratic regression*
		Measurement-Based Data Collection	*Measured the distances using a meterstick or measuring tape*
Bridge to the Classroom		*This activity could be done with Algebra 2 students as a motivating need to use matrices to solve systems of equations with matrices or to practice transforming the parent quadratic function. In Precalculus, this activity could be used with parametric equations, expressing the horizontal and vertical distances in terms of time.*	

Transparency 1: Ramp Setup



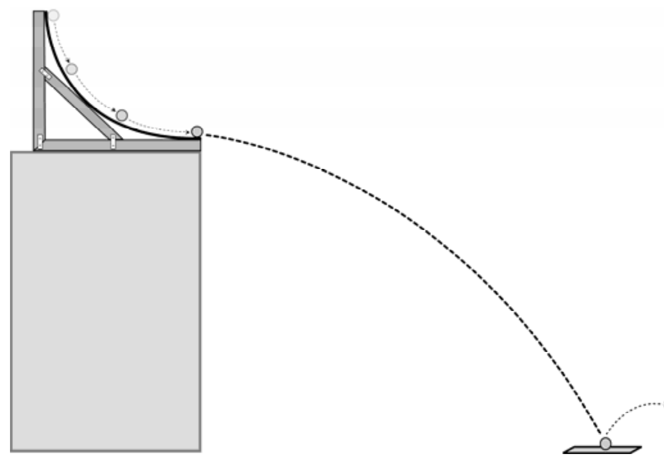
Transparency 2: Collecting the Third Data Point



Flying Off the Handle

In Hollywood movies, a classic way to end a car chase is to have a car drive off a cliff or into a canyon, with the hero jumping out of the car at the last minute to safety. Movie directors need to know exactly where the car will land so that they can have cameras in place to capture the motion on film. They also need to know where the car will be as it falls from the edge of the cliff to the floor of the canyon below so that they can have cameras in place there, too. How can we model this motion? How can we harness technology to apply this model to pinpoint the specific location of a moving object at any time?

To answer these questions, let's build a model that will enable us to simulate a car driving off a cliff. Use a wooden ramp and marble to simulate the car's motion. How would we develop a function rule to determine the placement of the cup on top of a stack of textbooks?



Set up the ramp on a high table. Roll the marble down the ramp and let it hit the floor to observe the motion of the marble.

1. Let the floor represent the *x*-axis and the end of the ramp be contained on the *y*-axis. Where would the origin of this coordinate system be?

2. In this coordinate system, what do *x* and *y* represent?

3. Consider the path of the marble. Based on your coordinate system, what does the *y*-intercept represent? What are the coordinates of the *y*-intercept? Record the coordinates as a point in the table.

Horizontal Distance (*x*)	Height of the Marble (*y*)

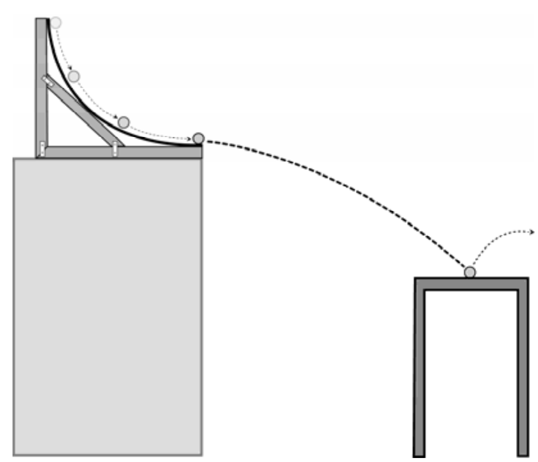
4. Based on your coordinate system, what does the *x*-intercept represent?

Roll the marble down the ramp and notice where it strikes the floor. Tape a piece of carbon paper on top of a piece of typing paper (carbon side down) where the marble touched the floor. Tape the paper to the floor.

Roll the marble down the ramp and let it strike the paper on the floor. Repeat at least twice so that you have data for at least 3 trials.

5. What are the coordinates of the *x*-intercept? Record the coordinates as a point in the table.

6. Place a chair or desk between the ramp and the point of impact on the floor. Repeat your data collection procedure to find the *x*- and *y*-coordinates of the point of impact on the chair or desk. Record your third data point in the table.



7. What kind of functional relationship do you think exists between the horizontal distance and the vertical distance of the marble?

8. Make a scatterplot of your data. Sketch your plot.

9. Use the coordinates of the three data points to write a function rule that could be used to predict the height of the marble, *y*, when it is a horizontal distance, *x*, from the ramp. Explain how you found your function.

10. Graph your function rule over your scatterplot and sketch your graph. Is the function rule a good fit? How can you tell? If not, how can you revise your function rule so that it is a better fit?

11. Place your cup on top of three textbooks. Where do you need to place the cup so that the marble will roll off the ramp and land inside the cup? Justify your choice.

12. Test your prediction. Was your prediction correct? Why or why not? If not, revise your prediction and test it again.

Flying Off the Handle: Intentional Use of Data

TEKS		
TEKS		
Question(s) to Pose to Students: Math		
Question(s) to Pose to Students: Tech		
Cognitive Rigor	Knowledge	
	Understanding	
	Application	
	Analysis	
	Evaluation	
	Creation	
Data Source(s)	Real-Time	
	Archival	
	Categorical	
	Numerical	
Setting	Computer Lab	
	Mini-Lab	
	One Computer	
	Graphing Calculator	
	Measurement-Based Data Collection	
Bridge to the Classroom		

Leader Notes: A Golden Idea

Explore/Explain Cycle II

Purpose:
Use a geometric context to generate data that can be modeled with an exponential function. Use technology to develop and analyze the exponential function.

Descriptor:
Participants will use Geometer's Sketchpad to examine a construction of a regular pentagon and a sequence of golden triangles. They will use angle relationships found in the pentagon and triangles created by its diagonals in order to make conjectures about similar triangles. Participants will use Geometer's Sketchpad to measure the lengths of the legs of successive isosceles triangles created by bisecting base angles. This data will be used to generate an exponential decay function. Participants will then dilate a golden isosceles triangle by a scale factor equal to the golden ratio. By measuring leg lengths of successive triangles, participants will gather data that will be used to generate an exponential growth function.

Duration:
2.5 hours

TEKS:

a(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

a(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

2A.1(B) collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

2A.4(A) Identify and sketch graphs of parent functions, including linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$)

functions, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = \frac{1}{x}$).

2A.4(B) Extend parent functions with parameters such as a in $f(x) = \frac{a}{x}$ and describe the effects of the parameter changes on the graph of parent functions.

2A.11(B) use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior.

2A.11(F) analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem.

G.5(A) Use numeric and geometric patterns to develop algebraic expressions representing geometric properties.

G.10(A) Use congruence transformations to make conjectures and justify properties of geometric figures including figures represented on a coordinate plane.

G.11(A) Use and extend similarity properties and transformations to explore and justify conjectures about geometric figures.

TAKS Objectives Supported by these Algebra 2 and Geometry TEKS:

- Objective 1: Functional Relationships
- Objective 2: Properties and Attributes of Functions
- Objective 6: Geometric Relationships and Spatial Reasoning
- Objective 8: Measurement and Similarity
- Objective 10: Mathematical Processes and Mathematical Tools

Technology:

- Internet access
- Graphing calculator
- Dynamic geometry software (such as Geometer's Sketchpad)
- Spreadsheet (such as Excel)

Materials:

Advanced Preparation: The Geometer's Sketchpad sketch **Golden Triangles.gsp** will need to be installed on each computer for participant use.

Presenter Materials: projector (computer or overhead) for graphing calculator

Per group: Geometer's Sketchpad sketch **Golden Triangles.gsp**

Per participant: graphing calculator, activity sheets

Leader Notes:
The golden ratio has been used in Western culture since the ancient Greeks used it to build the Parthenon and the Egyptians to build the Pyramids at Giza. It permeates Western art and architecture. The golden ratio is found in nature, including proportions in the human and other mammalian bodies, spiral shells, and insects.

Since the golden ratio is so prevalent, it carries with it algebraic implications as well. In this phase of the professional development, participants will use the golden ratio to collect data that can be modeled using an exponential function. They will use transformations to fit a function rule then use that function rule to make predictions.

Explore

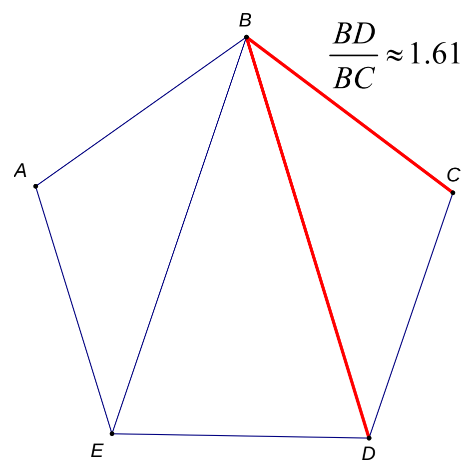
Posing the Problem:

Mathematics has been an important influence throughout civilization, from ancient times to the present. In ancient Egypt, Greece, and Rome, geometry and proportion were used in art and architecture. Medieval Europeans carried this tradition of using proportion as they built beautiful cathedrals. Renaissance painters and sculptors used proportion to convey their idea of natural beauty.

Where does this proportion originate? The Greeks found a certain ratio to be prevalent in the natural world around them.

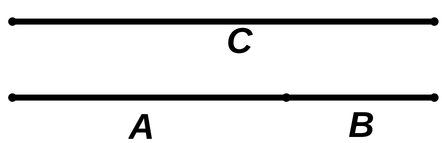
The golden ratio can be developed from several constructions. One way to construct the golden ratio is using the diagonals of a regular pentagon.

In regular pentagon *ABCDE* (shown at right), the ratio of the length of a diagonal from vertex B to the length of a side of the pentagon is always the same. This ratio is called the **golden ratio**, which the Greeks notated with the capital letter *phi*, or Φ.



$$\frac{BD}{BC} \approx 1.618 = \Phi$$

From a segment length perspective, the golden ratio is a geometric mean. Geometrically, segment (in the diagram below, of length *C*) can be split into two smaller segments (of lengths *A* and *B*).

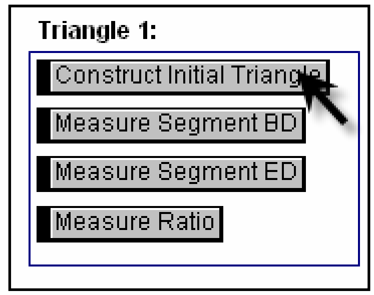


The splitting of the segment is such that the ratio of the length of the original segment to the length of the larger piece (*C*:*A*) is the same as the ratio of the length of the larger piece to the length of the smaller piece (*A*:*B*). In other words,

$$\frac{C}{A} = \frac{A}{B}$$

If the golden ratio is applied in succession to a geometric construction, what types of functional behaviors are present?

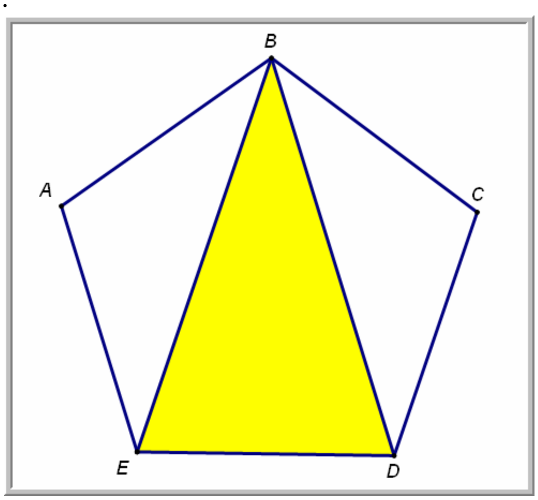
Part 1: Investigating Leg Length



Open the Geometer's Sketchpad sketch "Golden Triangles.GSP." If necessary, click on the "Investigating Leg Length" Tab. Pentagon *ABCDE* is a regular pentagon. From this regular pentagon, a series of triangles can be constructed. Click on the "Construct Initial Triangle" action button.

1. **What kind of triangle is ΔBED? How do you know?**

ΔBED is an isosceles triangle. Because ABCDE is a regular pentagon, $\overline{AB} \cong \overline{CB}$ and $\overline{AE} \cong \overline{CD}$. also, $m\angle A = 108°$ and $m\angle C = 108°$, so $\angle A \cong \angle C$. By SAS triangle congruence, $\Delta ABE \cong \Delta CBD$. Since corresponding parts of congruent triangles are congruent, $\overline{BE} \cong \overline{BD}$ and ΔBED is isosceles.



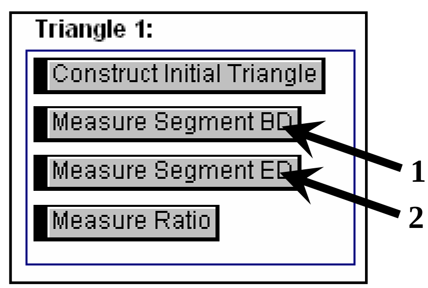
Facilitation Questions

- Do you see any congruent sides? How do you know they're congruent?
  Yes; all five sides of the pentagon are congruent because of the definition of a regular pentagon.
- What are the angle measures of the pentagon?
  In a regular pentagon, the interior angles all measure 108°.
- What is the sum of the measures of the interior angles of any triangle?
  180°
- What are the angle measures of the interior triangles?
  For ΔABE and ΔBCD the interior angles are 108°, 36°, and 36°. For ΔBED, the interior angles are 72°, 72°, and 36°.
- How could you use congruent triangles to show that some segments are congruent?
  Since corresponding parts of congruent triangles are congruent, if we can show that $\Delta ABE \cong \Delta CBD$*, then we can show that* $\overline{BE} \cong \overline{BD}$.

2. **Is your triangle classification from Question 1 true for every triangle formed when two diagonals are drawn from one vertex of a regular pentagon? How do you know?**
   Yes. For any regular pentagon, the diagonals from the same vertex will be congruent regardless of their length. Thus, the interior triangle will always be isosceles. (Incidentally, the other two triangles, in this case ABE and CBD, are also isosceles.)

Measure the length of $\overline{BD}$ **by clicking on the "Measure Segment BD" action button.**
Measure the length of $\overline{ED}$ **by clicking on the "Measure Segment ED" action button.**

3. **What is the ratio of the length of** $\overline{BD}$ **to the length of** $\overline{ED}$**? How did you find this ratio?**
   1.618, which can be found be either dividing BD by ED or by clicking on the Measure Ratio action button. If participants are fluent with The Geometer's Sketchpad, they may also be able to use the software to calculate the ratio.

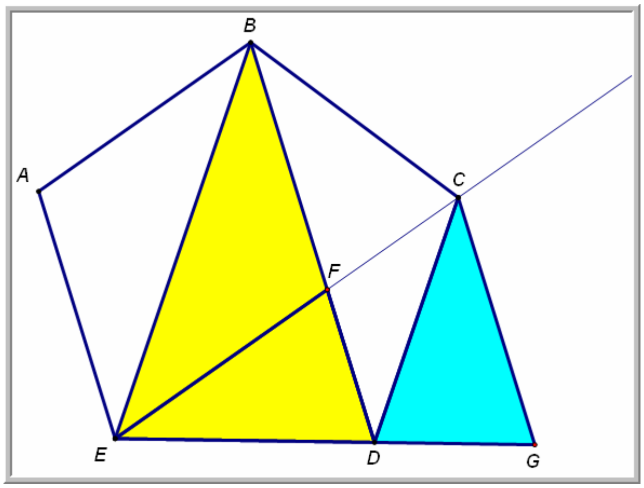


4. **What does this ratio represent?**
   1.618 is the golden ratio, Φ

Click on the "Construct Triangle 1" button. This animation bisects angle *BED*, then rotates the resulting triangle 108° to the same orientation as the original triangle. Measure the length of $\overline{CG}$ **by clicking on the "Measure Segment CG" button.**

5. What is the ratio of $\frac{BD}{CG}$? $\frac{CG}{BD}$? How do these numbers compare?

$\frac{BD}{CG} = \frac{12.33}{7.62} \approx 1.618$; $\frac{CG}{BD} = \frac{7.62}{12.33} \approx 0.618$; the ratios are reciprocals of each other.



6. How does ΔCDG compare to ΔBED? How do you know?

ΔCDG and ΔBED are similar triangles because their corresponding angles are congruent; by the AA similarity theorem, $\Delta CDG \sim \Delta BED$.

Facilitation Questions

- What does an angle bisector do?
  An angle bisector cuts an angle in half, into two smaller congruent angles.
- What do you need to know to show triangle congruence?
  Corresponding sides are congruent, corresponding angles are congruent. Side-side-side, side-angle-side, angle-side-angle, angle-angle-side are all possible combinations to show triangle congruence.
- What do you need to know to show triangle similarity?
  Corresponding angles are congruent, corresponding sides are proportional.
- Do you see any pairs of congruent angles?
  $\angle BED \cong \angle CDG$, $\angle EBD \cong \angle DCG$, $\angle BDE \cong \angle CGD$

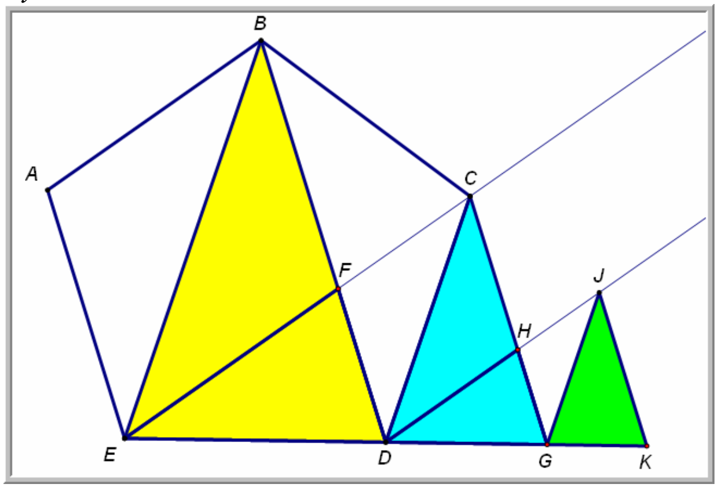
7. What scale factor could be applied to ΔBED to generate ΔCDG? Have you seen this ratio before? If so, where?

A scale factor of 0.618 was used. This number is the reciprocal of phi, the golden ratio.

Click the "Construct Triangle 2" button. This animation constructs ΔJGK in the same manner as the construction of ΔCDG. Measure the length of $\overline{JK}$ by clicking the "Measure Segment JK" button.

8. How does ΔJGK compare to ΔCDG? How do you know?

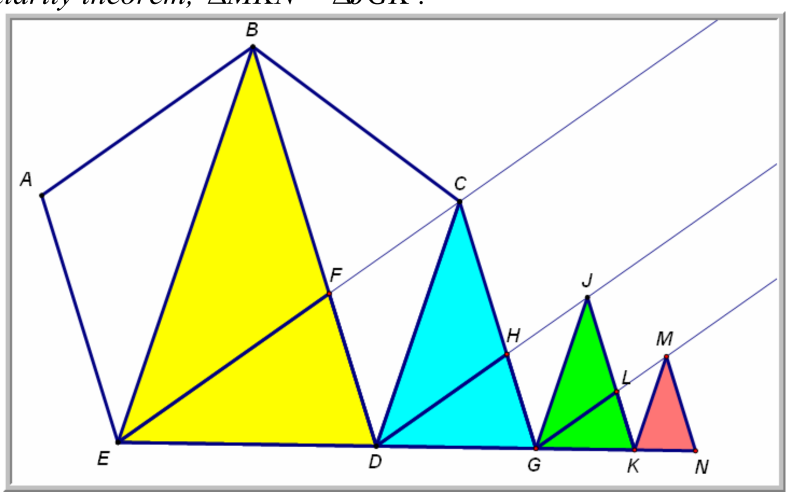
ΔJGK and ΔCDG are similar triangles because their corresponding angles are congruent; by the AA similarity theorem, $\Delta JGK \sim \Delta CDG$.



Click the "Construct Triangle 3" button. This animation constructs ΔMKN in the same manner as the construction of ΔJGK. Measure the length of $\overline{MN}$ by clicking the "Measure Segment MN" button.

9. How does ΔMKN compare to ΔJGK? How do you know?

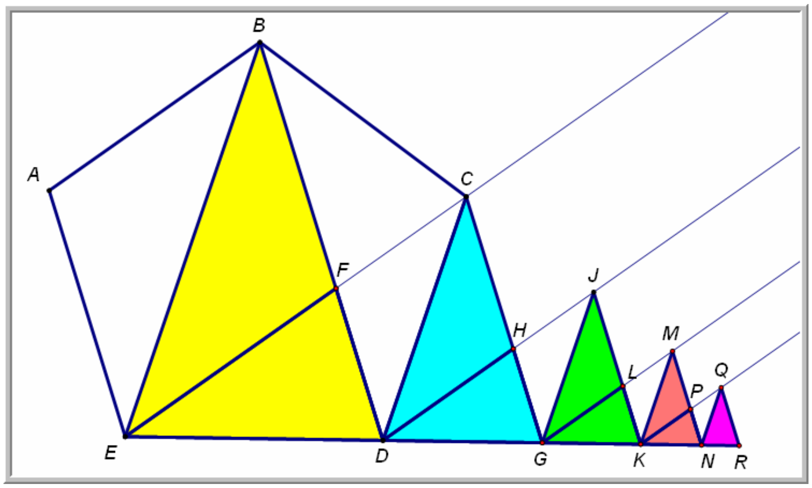
ΔMKN and ΔJGK are similar triangles because their corresponding angles are congruent; by the AA similarity theorem, $\Delta MKN \sim \Delta JGK$.



Click the "Construct Triangle 4" button. This animation constructs ΔQNR in the same manner as the construction of ΔMKN. Measure the length of $\overline{QR}$ by clicking the "Measure Segment QR" button.

10. How does ΔQNR compare to ΔMKN? How do you know?

ΔQNR and ΔMKN are similar triangles because their corresponding angles are congruent; by the AA similarity theorem, $\Delta QNR \sim \Delta MKN$.



11. What patterns do you observe in the sequence of triangles?

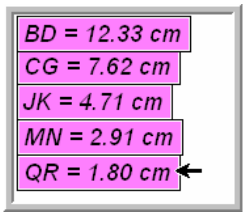
Each triangle is created by taking the angle bisector of the previous triangle and rotating it about a point. Investigation of segment lengths reveals that all of the triangles are similar.

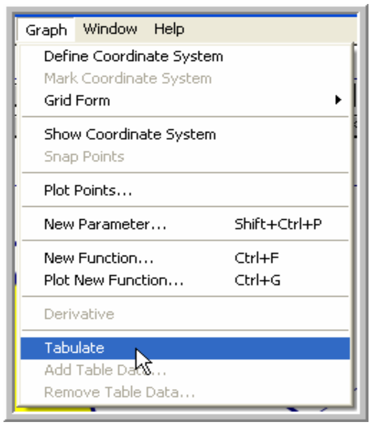
12. Record the measures of the leg of each triangle in the following table.

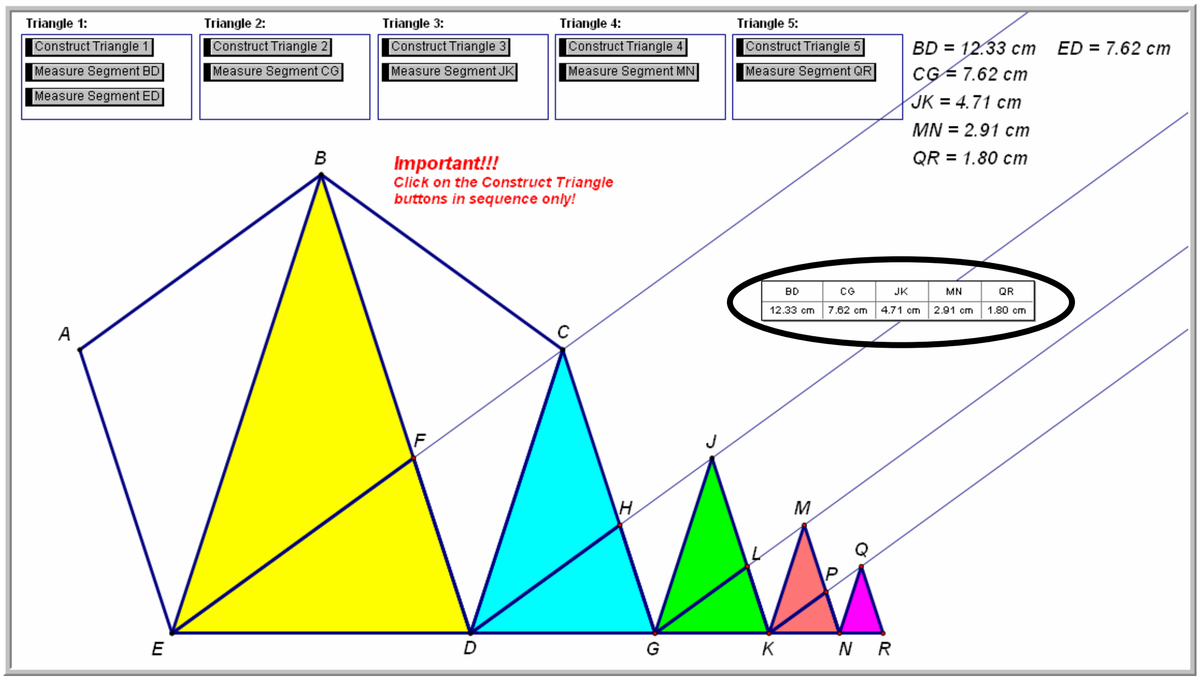
Note: Sample answers appear in the table below. Participants' actual measures will vary depending on the screen resolution and the settings in Geometer's Sketchpad. All of the sample answers that are generated from data that participants will collect are generated from this data set.

Triangle		Name of Leg	Length of Leg	Process	Ratio
Name	#				
ΔBED	0	BD	12.33		
ΔCDG	1	CG	7.62	$\frac{CG}{BD} = \frac{7.62}{12.33}$	$\frac{CG}{BD} = 0.618$
ΔJGK	2	JK	4.71	$\frac{JK}{CG} = \frac{4.71}{7.62}$	$\frac{JK}{CG} = 0.618$
ΔMKN	3	MN	2.91	$\frac{MN}{JK} = \frac{2.91}{4.71}$	$\frac{MN}{JK} = 0.618$
ΔQNR	4	QR	1.80	$\frac{QR}{MN} = \frac{1.80}{2.91}$	$\frac{QR}{MN} = 0.618$

Technology Tip: Once participants have measured the length of each leg, if they select each measurement then choose Tabulate from the Graph menu, Geometer's Sketchpad will create a table as shown in the figure. Then, participants can copy the data into the table readily.

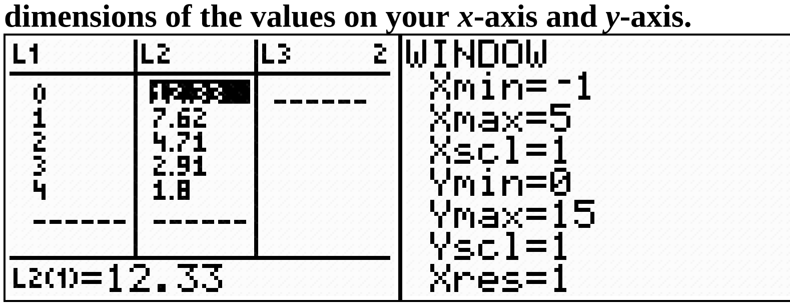


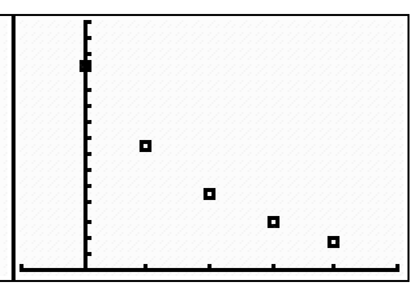




13. Record the ratio of each leg length to its previous leg length in the table.

14. Use an appropriate technology to generate a scatterplot of Leg Length vs. Triangle Number (let ΔBED be Triangle Number 0). Sketch your scatterplot and indicate the dimensions of the values on your x-axis and y-axis.





15. Based on your scatterplot, what type of function would model the relationship found in the data? Justify your choice.

Answers may vary. Participants should notice a non-linear decreasing curve. An exponential decay model might model the data set well.

> **Facilitation Questions**
> - What parameters does your parent function have?
>   *Answers may vary. Exponential functions are generally of the form $y = ab^x$.*
> - What do these parameters represent?
>   *Answers may vary. For exponential functions of the form $y = ab^x$, a represents the initial value (when $x = 0$) and b represents the constant ratio between consecutive y-values.*

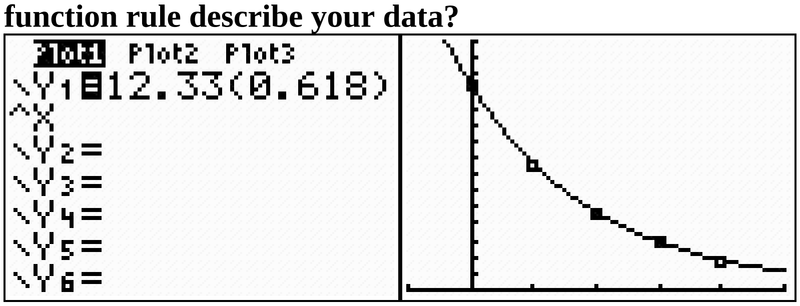
16. Use the parent function from Question 15 to determine a function rule to describe the relationship between triangle number and leg length. What do the variables in your function rule represent? What do the constants in your function rule represent?

Sample function rule: $y = 12.33(0.618)^x$

Let x represent the triangle number and y represent the length of the leg of that isosceles triangle. 12.33 represents the leg length of the initial triangle, which was constructed from the pentagon in this geometric sequence. The exponential base, 0.618, is the successive ratio of 0.618 which is the reciprocal of phi, or $\frac{1}{\Phi}$. *The base represents the rate of dilation in this geometric sequence. Since it is less than 1, this function represents an exponential decay.*

> **Facilitation Questions**
> - Are the data points increasing or decreasing?
>   *decreasing*
> - Are the data points increasing or decreasing at a constant rate?
>   *No. The rate of decrease slows as the triangle number gets larger.*

17. Graph your function rule over the scatterplot. Sketch your graph. How well does the function rule describe your data?



The function rule smoothly connects all five data points indicating a good fit for this data set.

18. Compare the domain of your data and the domain of the function rule.

The domain of the data set is {0, 1, 2, 3, 4} and the domain of the function is all real numbers or $\{x : x \in \Re\}$. *The domain of the data set is a discrete subset of the domain of the function.*

19. Compare the range of your data and the range of the function rule.

The range of the data set is {1.8, 2.91, 4.71, 7.62, 12.33}. The range of the function is $\{y : y \in \Re, y > 0\}$. *The range of the data set is a discrete subset of the range of the function.*

20. What will be the length of the leg of the 9th triangle in this sequence? Explain how you determined your answer.

For the 9th triangle, x = 9. Use the Table feature of a graphing calculator to find the y-value when x = 9.

X	Y1
5	1.1115
6	.6869
7	.4245
8	.26234
9	.16213
10	.1002
11	.06192

X=9

21. Which triangle will be the first one to have a leg length less than 0.5 cm? Explain how you determined your answer.

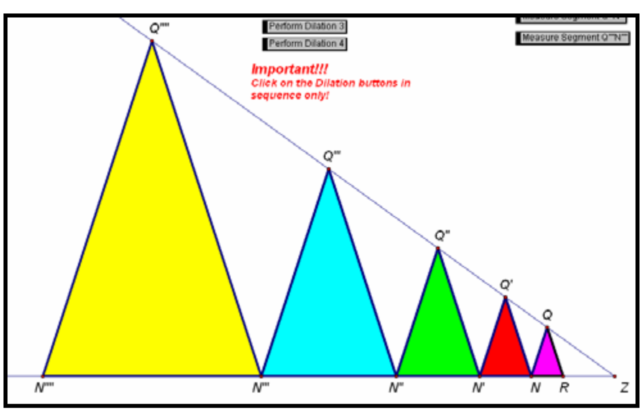
A leg length of 0.5 cm has a y-value of 0.5 in our function rule. Symbolically, this inequality is $0.5 < 12.23(0.618)^x$. *Use the Table feature of the graphing calculator to find the first x-value that satisfies the inequality. The 7th triangle in the series is the first one to have a leg length of less than 0.5 cm.*

X	Y1
5	1.1115
6	.6869
7	.4245
8	.26234
9	.16213
10	.1002
11	.06192

X=7

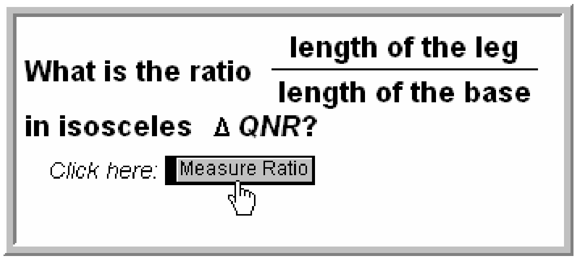
Part 2: Investigating Dilations

In this part, participants will click to a new page in the Geometer's Sketchpad sketch, Golden Triangles.gsp. This time, we will start with ΔQNR, which was the last triangle constructed in the previous sketch. Recall that ΔQNR is a golden isosceles triangle.

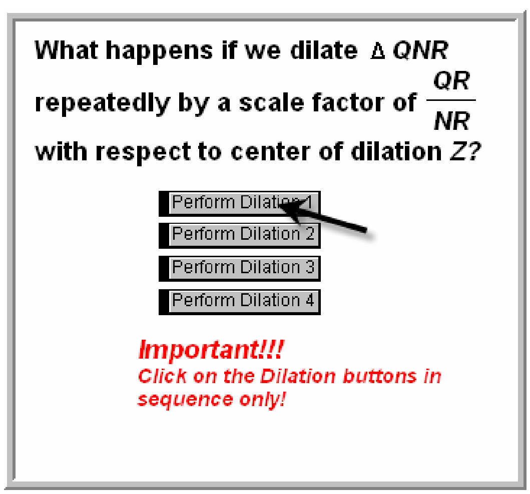


In the previous activity, you constructed a series of golden isosceles triangles. What happens if we take a golden triangle and enlarge it repeatedly?

1. **In the same Geometer's Sketchpad sketch, click on the "Investigating Dilations" tab. In isosceles ΔQNR, what is the ratio of the length of the leg, *QR*, to the length of the base, *NR*?**
   Participants may obtain their answer by clicking the "Measure Ratio" button in the top left corner of the sketch. The ratio is 1.618, which is phi, the golden ratio.



2. **Click the "Perform Dilation 1 button." Describe what you see.**
   $\overrightarrow{ZQ}$ *and* $\overrightarrow{ZN}$ *appear.* ΔQNR *dilates along these rays by a scale factor of* $\frac{QR}{NR} \approx 1.618 = \Phi$ *creating* $\Delta Q'N'N$.



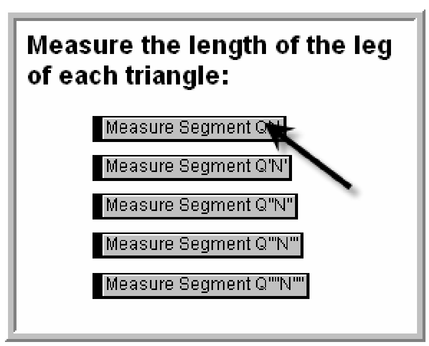
3. **Click the remaining "Perform Dilation" buttons in sequential order, one at a time. Describe the result.**
   Three more triangles are generated, each one by a scale factor of $\frac{QR}{NR} \approx 1.618 = \Phi$ *from the previous triangle.*

4. How do each of the triangles compare with each other? How do you know?
Each of the five triangles are similar to each other. Their side lengths are all proportional by the same scale factor, $\frac{QR}{NR} \approx 1.618 = \Phi$.

Facilitation Questions

- What geometric properties does a dilation have?
  A dilation is a proportional enlargement or reduction. Thus, the figure is enlarged or reduced in such a way that the side lengths of the image are proportional to the side lengths of the preimage.
- Which corresponding sides are congruent?
  No corresponding sides are congruent. They are proportional by a scale factor of 1.618, or Φ.
- Which corresponding angles are congruent?
  Three sets of corresponding angles are congruent: the set of vertex angles for each isosceles triangle and both sets of base angles.

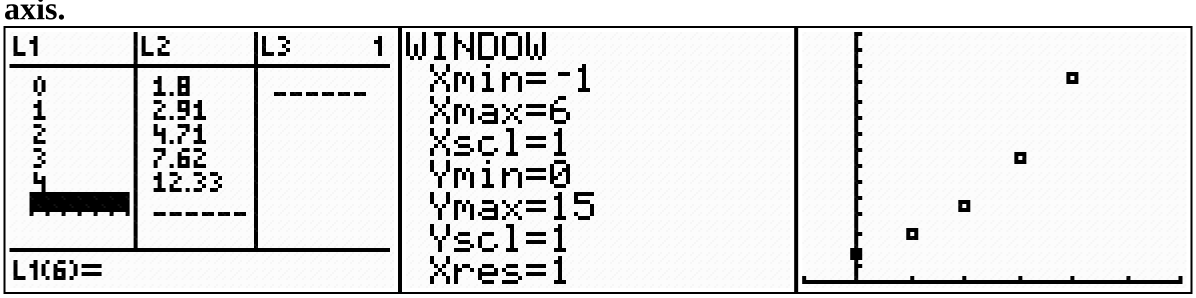
5. Measure the leg lengths of each triangle by clicking the "Measure Segment" buttons in order, one at a time. Record the segment lengths in the table below.



Note: Sample answers appear in the table below. Participants' actual measures will vary depending on the screen resolution and the settings in Geometer's Sketchpad. This data set generates all of the sample answers generated from data that participants will collect.

Triangle		Name of Leg	Length of Leg	Process	Ratio
Name	Dilation Number				
ΔQNR	*0*	QN	*1.80*		
$\Delta Q'N'N$	*1*	$Q'N'$	*2.91*	$\frac{Q'N'}{QN}=\frac{2.91}{1.80}\approx 1.618$	$\frac{Q'N'}{QN}=1.618$
$\Delta Q''N''N'$	*2*	$Q''N''$	*4.71*	$\frac{Q''N''}{Q'N'}=\frac{4.71}{2.91}\approx 1.618$	$\frac{Q''N''}{Q'N'}=1.618$
$\Delta Q'''N'''N''$	*3*	$Q'''N'''$	*7.62*	$\frac{Q'''N'''}{Q''N''}=\frac{7.62}{4.71}\approx 1.618$	$\frac{Q'''N'''}{Q''N''}=1.618$
$\Delta Q''''N''''N'''$	*4*	$Q''''N''''$	*12.33*	$\frac{Q''''N''''}{Q'''N'''}=\frac{12.33}{7.62}\approx 1.618$	$\frac{Q''''N''''}{Q'''N'''}=1.618$

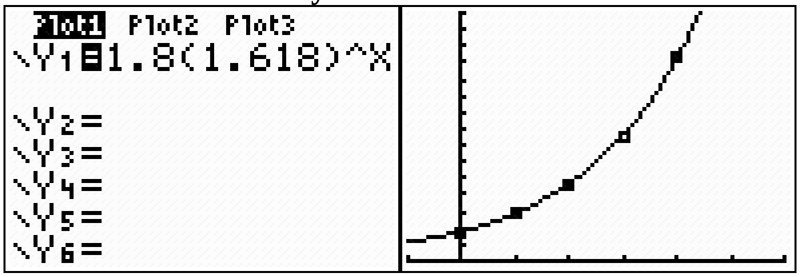
6. **Record the successive ratios in the appropriate column of your table. Use an appropriate technology to generate a scatterplot of Leg Length vs. Dilation Number. Sketch your scatterplot and indicate the dimensions of the values on your *x*-axis and *y*-axis.**



7. **Based on your scatterplot, what type of function would model the relationship found in the data? Justify your choice.**
   Answers may vary. Participants should notice a non-linear increasing curve. An exponential growth model might model the data set well.

8. **Use the parent function from Question 7 to determine a function rule to describe the relationship between dilation number and leg length. What do the variables in your function rule represent? What do the constants in your function rule represent?**
   Sample function rule: $y = 1.80(1.618)^x$
   Let x represent the dilation number and y represent the length (in centimeters) of the leg of that isosceles triangle. 1.80 represents 1.80 cm, the leg length of the initial triangle. The exponential base, 1.618, is the scale factor of dilation which is the golden ratio, Φ.

9. Graph your function rule over the scatterplot. Sketch your graph. How well does the function rule describe your data?



The function rule smoothly connects all five data points indicating a good fit for this data set.

10. What scale factor could be used to generate the second dilation from the original triangle without generating the first dilation?

Φ^2

The first dilation is generated by a scale factor of Φ. To generate the second dilation, we multiply again by a scale factor of Φ. Multiplying the original dimensions by Φ × Φ is the same as multiplying by Φ^2.

Facilitation Questions

- What does a dilation do to the side lengths of a figure?
  A dilation multiplies the side lengths of a preimage by a scale factor.
- Arithmetically, how can we notate repeated multiplication?
  Using exponents

11. What scale factor could be used to generate the third dilation from the original triangle without generating the first two dilations?

Φ^3

Since each successive dilation is created by another multiplication of the dimensions by Φ, a third dilation will multiply the original dimensions by a factor of Φ^3.

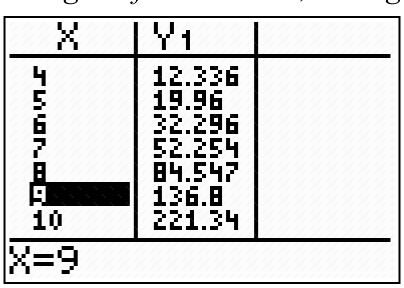
12. How could you predict the scale factor in terms of the dilation number?

Raise Φ to the power of the dilation number.

13. What scale factor would be used to generate the 9th dilation in the sequence? What would the leg length of this triangle be? Explain how you determined your answer.

Φ^9

Using the function rule, the leg length would be $y = 1.80(\Phi)^9 = 1.80(1.618)^9 \approx 136.8\ cm$.



X	Y1
4	12.336
5	19.96
6	32.296
7	52.254
8	84.547
9	136.8
10	221.34

X=9

14. Which dilation will be the first one to have a leg length of at least 2.5 meters? Explain how you determined your answer.

A leg length of 2.5 meters has a y-value of 250 cm in our function rule. Symbolically, this inequality is $2.50 \geq 1.80(1.618)^x$. *Use the Table feature of the graphing calculator to find the first x-value that satisfies the inequality. The 11th triangle in the series is the first one to have a leg length of less at least 2.5 meters.*

X	Y1	
6	32.296	
7	52.254	
8	84.547	
9	136.8	
10	221.34	
11	358.13	
12	579.45	

X=11

Explain

This phase of the training should be a whole group discussion. Pose the following questions to participants one at a time, allowing enough time for meaningful discourse to take place about each question.

In this phase, use the debrief questions to prompt participant groups to discuss their responses to the data analysis. At this stage in the professional development, participants should be familiar with using the graphing calculator and a spreadsheet. If none of the participant groups use one of these methods, ask them how they could have used that method to analyze the data. This information is important to the discussion of relative advantages and disadvantages of different types of technology. The reasons that a participant group did not choose a particular technology are as important (if not more so) than the justifications a group gives for the technology that they did choose.

1. **How did you develop your scatterplots? Why did you choose this method?**
   Ask participants to share their methods and their reasons for making that choice. If none of the participants choose one of the technologies (graphing calculator or spreadsheet), ask participants why no one made that choice.

   See "Technology Tutorial: A Golden Idea" for detailed instructions.

2. **How did you develop your function rules? Why did you choose this method?**
   Ask participants to share their methods and their reasons for making that choice. If none of the participants chooses one of the technologies (graphing calculator or spreadsheet), ask participants why no one made that choice.

   See "Technology Tutorial: A Golden Idea" for detailed instructions.

3. **In what ways are the domain and range for the situation and the domain and range for the function rule used to model the situations?**
The domain and range for the situation are each subsets of the domain and range of the function rules, respectively.

4. **How were expressions evaluated in this exploration?**
When given triangle or dilation number, participants were asked to find the leg length.

5. **How were equations solved in this exploration?**
When given a leg length participants were asked to find a triangle or dilation number.

6. **How are the bases of the two exponential functions from Part 1 and Part 2 related?**
In Part 1, the exponential decay function had a base of 0.618, or $\frac{1}{\Phi}$*. In Part 2, the exponential growth function had a base of 1.618, or* Φ*. They are reciprocals of one another.*

Note to Leader: *Record or have a participant volunteer record the responses to Questions 6 and 7 on chart paper to use in the Elaborate phase of the professional development.*

7. **What are the relative advantages and disadvantages of using a graphing calculator to solve this problem?**
Responses may vary.

The data analysis can be done in a few keystrokes. The power to set your own parameters and graph the function rule empowers the participant to use numerical analysis to calculate meaningful parameters such as a constant of variation. The graphical analysis features of the calculator make it easy to use the graph to solve problems by tracing and calculating the intersection of lines.

However, the small screen is difficult to see, and the axes in the window cannot be labeled.

8. **What are the relative advantages and disadvantages of using a spreadsheet to solve this problem?**
Responses may vary.

The regression equation is calculated quickly on the spreadsheet. The axes can be clearly labeled with numbers and text labels. Labeled axes help the participant to use the graph to estimate solutions to problems that can be solved graphically. The graph can be enlarged or reduced then copied and pasted into other computer documents such as a Word or PowerPoint document to communicate the solution to a problem.

However, the participant is limited to the regression equations available in the spreadsheet. There are no graphical analysis features in most spreadsheets, so only estimates rather than exact solutions can be obtained graphically.

9. **Does the use of technology in this exploration reinforce pencil and paper symbolic algebraic manipulation? If so how? If not, what questions need to be asked so that pencil and paper symbolic algebraic manipulation is reinforced?**
   Answers will vary. The point of this question is to stimulate discourse as to the importance of teacher questioning regardless of the environment that students are working in not to evaluate the use of technology or pencil and paper procedures.

10. **What TEKS does this activity address?**
    Participants should brainstorm a list of TEKS that they believe they have covered in this activity. The Leader Notes contain a comprehensive list of the TEKS addressed in this phase of the professional development. If participants do not mention some of these TEKS, then ask them how the activity also covers them. Since this activity also incorporates Geometry TEKS that are assessed on 11th Grade Exit Level TAKS, this is a good opportunity to discuss the integration of Geometry and Algebra 2 TEKS.

11. **How does the technology that you used enhance the teaching of those TEKS?**
    Responses may vary. However, participants should note that using technology enables them to explore a mathematical concept to a much deeper level. For example, in this activity, using a spreadsheet or the List Editor in a graphing calculator allows participants to make quick computations that allow them to determine the constant multiplier via successive quotients.

    Additionally, the technology of the Geometer's Sketchpad enables participants to quickly and easily generate data in a geometric context that can be used to explore functional relationships. The advantage to using a geometric context is that it affords Algebra 2 teachers an opportunity to review skills that are tested in the geometry objectives on TAKS while teaching Algebra 2 TEKS at the same time.

A Golden Idea: Intentional Use of Data

1. At the close of *A Golden Idea*, distribute the **Intentional Use of Data** activity sheet to each participant.

2. Prompt the participants to work in pairs to identify those TEKS that received greatest emphasis during this activity. Also prompt the participants to identify two key questions that are emphasized during this activity. Allow four minutes for discussion.

Facilitation Questions
- Which mathematics TEKS form the primary focus of this activity?
- What additional mathematics TEKS support the primary TEKS?
- How do these TEKS translate into guiding questions to facilitate student exploration of the content?
- How do your questions reflect the depth and complexity of the TEKS?
- How do your questions support the use of technology?

3. As a whole group, discuss responses for two to three minutes.

4. As a whole group, identify the level(s) of rigor (based on Bloom's taxonomy) addressed, the types of data, the setting, and the data sources used during this Explore/Explain cycle. Allow three minutes for discussion.

Facilitation Question
- What attributes of the activity support the level of rigor that you identified?

5. As a whole group, discuss how this activity might be implemented in other settings. Allow five minutes for discussion.

6. Prompt the participants to set aside the completed Intentional Use of Data activity sheet for later discussion. These completed activity sheets will be used during the Elaborate phase as prompts for generating attributes of judicious users of technology.

Facilitation Questions
- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) per participant?
- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) per small group of participants?
- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) for the entire group of participants?
- How would this activity change if we had used computer-based applications instead of graphing calculators?
- How might we have made additional use of available technologies during this activity?
- How does technology enhance learning?

Sample responses:

TEKS		*a(5), a(6), 2A.1B, 2A.11B, 2A.11F*	
		G.5A, G.10A, G.11A	
Question(s) to Pose to Students	**Math**	*What other situations can you think of that could be modeled with an exponential function?* *What patterns did you discover? What other patterns could there be?*	
	Tech	*How did the dynamic geometry software help you collect data?* *How did the graphing calculator or Excel help you analyze the data?*	
Cognitive Rigor		Knowledge	√
		Understanding	√
		Application	√
		Analysis	√
		Evaluation	√
		Creation	√
Data Source(s)		Real-Time	*Yes; using Geometer's Sketchpad generated on-the-spot data*
		Archival	
		Categorical	
		Numerical	
Setting		Computer Lab	*Each student uses the computer.*
		Mini-Lab	*In groups students take turns or groups switch out.*
		One Computer	*A student operates the control as other students read directions, entire class records data.*
		Graphing Calculator	*Could be used to enter data and find relationships.*
		Measurement-Based Data Collection	*Could be done at stations or individually.*
Bridge to the Classroom		*This activity transfers directly to the classroom with the only modifications being the settings addressed above.*	

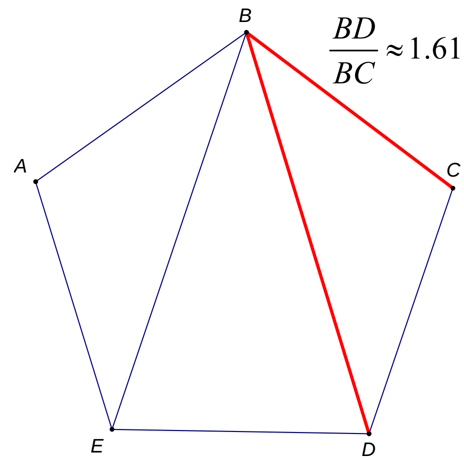
A Golden Idea

Mathematics has been an important influence throughout civilization, from ancient times to the present. In ancient Egypt, Greece, and Rome, geometry and proportion were used in art and architecture. Medieval Europeans carried this tradition of using proportion as they built beautiful cathedrals. Renaissance painters and sculptors used proportion to convey their idea of natural beauty.

Where does this proportion originate? The Greeks found a certain ratio to be prevalent in the natural world around them.

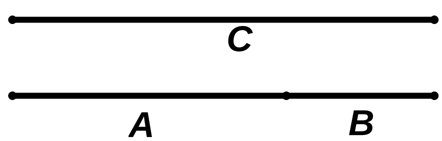
The golden ratio can be developed from several constructions. One way to construct the golden ratio is using the diagonals of a regular pentagon.

In regular pentagon *ABCDE* (shown at right), the ratio of the length of a diagonal from vertex B to the length of a side of the pentagon is always the same. This ratio is called the **golden ratio**, which the Greeks notated with the capital letter *phi*, or Φ.



$$\frac{BD}{BC} \approx 1.618 = \Phi$$

From a segment length perspective, the golden ratio is a geometric mean. Geometrically, segment (in the diagram below, of length *C*) can be split into two smaller segments (of lengths *A* and *B*).

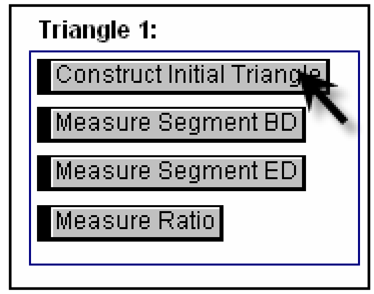


The splitting of the segment is such that the ratio of the length of the original segment to the length of the larger piece (*C*:*A*) is the same as the ratio of the length of the larger piece to the length of the smaller piece (*A*:*B*). In other words,

$$\frac{C}{A} = \frac{A}{B}$$

If the golden ratio is applied in succession to a geometric construction, what types of functional behaviors are present?

Part 1: Investigating Leg Length

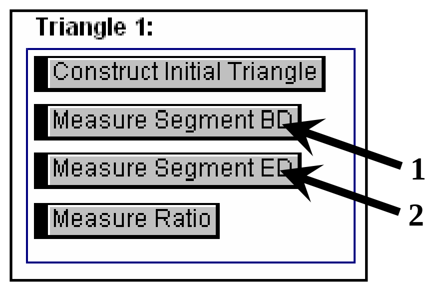


Open the Geometer's Sketchpad sketch "Golden Triangles.GSP." Pentagon *ABCDE* is a regular pentagon. From this regular pentagon, a series of triangles can be constructed. Click on the "Construct Initial Triangle" action button.

1. What kind of triangle is ΔBED? How do you know?

2. Is your triangle classification from Question 1 true for every triangle formed when two diagonals are drawn from one vertex of a regular pentagon? How do you know?

Measure the length of $\overline{BD}$ by clicking on the "Measure Segment BD" action button. Measure the length of $\overline{ED}$ by clicking on the "Measure Segment ED" action button.



3. What is the ratio of the length of $\overline{BD}$ to the length of $\overline{ED}$? How did you find this ratio?

4. What does this ratio represent?

Click on the "Construct Triangle 1" button. This animation bisects angle *BED*, then rotates the resulting triangle 108° to the same orientation as the original triangle. Measure the length of $\overline{CG}$ by clicking on the "Measure Segment CG" button.

5. What is the ratio of $\frac{BD}{CG}$? $\frac{CG}{BD}$? How do these numbers compare?

6. How does ΔCDG compare to ΔBED? How do you know?

7. What scale factor could be applied to ΔBED to generate ΔCDG? Have you seen this ratio before? If so, where?

Click on the "Construct Triangle 2" button. This animation constructs ΔJGK in the same manner as the construction of ΔCDG. Measure the length of $\overline{JK}$ by clicking on the "Measure Segment JK" button.

8. How does ΔJGK compare to ΔCDG? How do you know?

Click the "Construct Triangle 3" button. This animation constructs ΔMKN in the same manner as the construction of ΔJGK. Measure the length of $\overline{MN}$ by clicking the "Measure Segment MN" button.

9. How does ΔMKN compare to ΔJGK? How do you know?

Click the “Construct Triangle 4” button. This animation constructs ΔQNR in the same manner as the construction of ΔMKN. Measure the length of $\overline{QR}$ by clicking the “Measure Segment QR” button.

10. How does ΔQNR compare to ΔMKN? How do you know?

11. What patterns do you observe in the sequence of triangles?

12. Record the measures of the leg of each triangle in the following table.

Triangle		Name of Leg	Length of Leg	Process	Ratio
Name	**#**				
ΔBED					
ΔCDG					$\frac{CG}{BD} =$
ΔJGK					$\frac{JK}{CG} =$
ΔMKN					$\frac{MN}{JK} =$
ΔQNR					$\frac{QR}{MN} =$

13. Record the ratio of each leg length to its previous leg length in the table.

14. Use an appropriate technology to generate a scatterplot of Leg Length vs. Triangle Number (let ΔBED be Triangle Number 0). Sketch your scatterplot and indicate the dimensions of the values on your *x*-axis and *y*-axis.

15. Based on your scatterplot, what type of function would model the relationship found in the data? Justify your choice.

16. Use the parent function from Question 15 to determine a function rule to describe the relationship between triangle number and leg length. What do the variables in your function rule represent? What do the constants in your function rule represent?

17. Graph your function rule over the scatterplot. Sketch your graph. How well does the function rule describe your data?

18. Compare the domain of your data and the domain of the function rule.

19. Compare the range of your data and the range of the function rule.

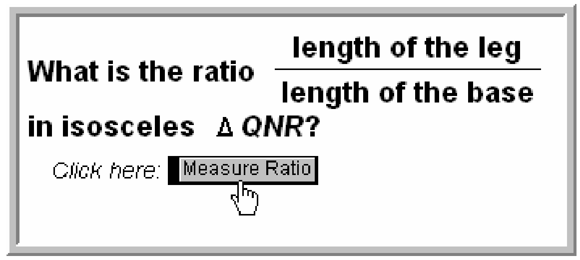
20. What will be the length of the leg of the 9th triangle in this sequence? Explain how you determined your answer.

21. Which triangle will be the first one to have a leg length less than 0.5 cm? Explain how you determined your answer.

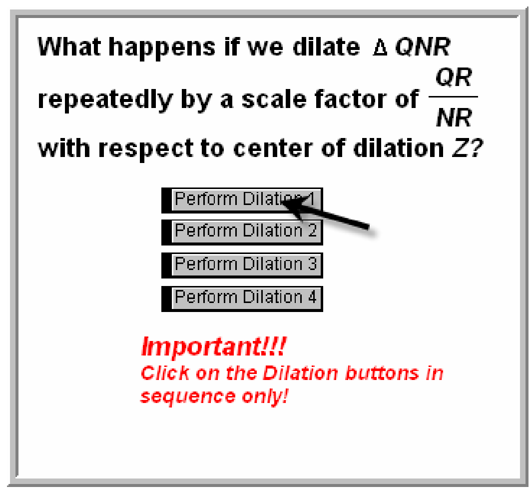
Part 2: Investigating Dilations

In the previous activity, you constructed a series of golden isosceles triangles. What happens if we take a golden triangle and enlarge it repeatedly?

1. In the same Geometer's Sketchpad sketch, click on the "Investigating Dilations" tab. In isosceles ΔQNR, what is the ratio of the length of the leg, QR, to the length of the base, NR?



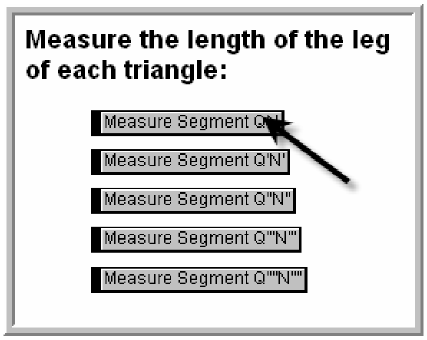
2. Click the "Perform Dilation 1 button." Describe what you see.



3. Click the remaining "Perform Dilation" buttons in sequential order, one at a time. Describe the result.

4. How do each of the triangles compare with each other? How do you know?

5. Measure the leg lengths of each triangle by clicking the "Measure Segment" buttons in order, one at a time. Record the segment lengths in the table below.



Triangle		**Name of Leg**	**Length of Leg**	**Process**	**Ratio**
Name	**Dilation Number**				
ΔQNR	*0*				
$\Delta Q'N'N$	*1*				$\frac{Q'N'}{QN} =$
$\Delta Q''N''N'$	*2*				$\frac{Q''N''}{Q'N'} =$
$\Delta Q'''N'''N''$	*3*				$\frac{Q'''N'''}{Q''N''} =$
$\Delta Q''''N''''N'''$	*4*				$\frac{Q''''N''''}{Q'''N'''} =$

6. Record the successive ratios in the appropriate column of your table. Use an appropriate technology to generate a scatterplot of Leg Length vs. Dilation Number. Sketch your scatterplot and indicate the dimensions of the values on your *x*-axis and *y*-axis.

7. Based on your scatterplot, what type of function would model the relationship found in the data? Justify your choice.

8. Use the parent function from Question 7 to determine a function rule to describe the relationship between dilation number and leg length. What do the variables in your function rule represent? What do the constants in your function rule represent?

9. Graph your function rule over the scatterplot. Sketch your graph. How well does the function rule describe your data?

10. What scale factor could be used to generate the second dilation from the original triangle without generating the first dilation?

11. What scale factor could be used to generate the third dilation from the original triangle without generating the first two dilations?

12. How could you predict the scale factor in terms of the dilation number?

13. What scale factor would be used to generate the 9th dilation in the sequence? What would the leg length of this triangle be? Explain how you determined your answer.

14. Which dilation will be the first one to have a leg length of at least 2.5 meters? Explain how you determined your answer.

A Golden Idea: Intentional Use of Data

TEKS			
Question(s) to Pose to Students	**Math**		
	Tech		
Cognitive Rigor	Knowledge		
	Understanding		
	Application		
	Analysis		
	Evaluation		
	Creation		
Data Source(s)	Real-Time		
	Archival		
	Categorical		
	Numerical		
Setting	Computer Lab		
	Mini-Lab		
	One Computer		
	Graphing Calculator		
	Measurement-Based Data Collection		
Bridge to the Classroom			

Leader Notes: I've Seen the Light!

Explore/Explain Cycle III

Purpose:

Solve a problem by collecting and analyzing data leading to the use of a rational function as a model. Create numerical and graphical representations to analyze the data using technology.

Descriptor:

Participants will explore the relationship between the intensity of light and the distance from the light source by using a light sensor and calculator-based laboratory to collect data. They will create tabular and graphical representations using a graphing calculator, spreadsheet, and TI-Interactive. Participants will compare and contrast the use of these technologies and their effectiveness in representing the data and communicating the results of the data analysis.

Duration:

2 hours

TEKS:

a(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

a(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

2A.1(B) collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

2A.4(A) Identify and sketch graphs of parent functions, including linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$) functions, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = \frac{1}{x}$).

2A.4(B) Extend parent functions with parameters such as a in $f(x) = \frac{a}{x}$ and describe the effects of the parameter changes on the graph of parent functions.

2A.10(B)	analyze various representations of rational functions with respect to problem situations;
2A.10(C)	determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities;
2A.10(D)	determine the solutions of rational equations using graphs, tables, and algebraic methods;
2A.10(E)	determine solutions of rational inequalities using graphs and tables;
2A.10(F)	analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem; and
2A.10(G)	use functions to model and make predictions in problem situations involving direct and inverse variation.

TAKS Objectives Addressed by these Algebra 2 TEKS:

- Objective 1: Functional Relationships
- Objective 2: Properties and Attributes of Functions
- Objective 10: Mathematical Processes and Mathematical Tools

Technology:

- CBL2 or other calculator-based data collection device
- Light probe or sensor
- Graphing Calculator
- Spreadsheet
- TI-Interactive
- Graph linking capability, such as TI-Connect or Casio Program-Link

Materials:

Advanced Preparation:	Be sure that the flashlight batteries are fresh. Have extra batteries for the CBL2 on hand.
Presenter Materials:	projector for graphing calculator and computer demonstration
Per group:	CBL2, light probe, flashlight with fresh batteries, 2 or 3 meter sticks OR metric tape measure, graph link cable appropriate to type of calculator being used
Per participant:	graphing calculator, activity sheets

Leader Notes:

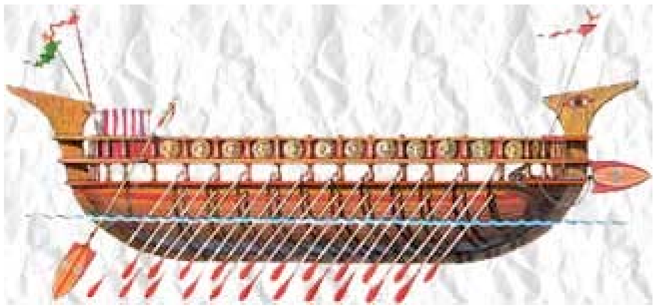
If you shine a flashlight at a far wall in a dark room, the light will spread out and diffuse over most of the wall. As you get closer to the wall, the light covers a smaller area but is brighter and more clearly defined. The brightness of the light is called its intensity and is measured in watts per unit area, usually square meters or square centimeters.

The relationship is an inverse-square variation relationship of the form $y = \frac{k}{x^2}$, which is one form of rational function. Participants will be asked to gather the data and analyze it on their own. During the debrief of the Explain phase, several ways of analyzing the data will be discussed and participants will be asked to identify comparative advantages and disadvantages of each method.

Participants will be using the CBL2 or equivalent calculator-based laboratory data collection device. Participants' instructions for using the light probe are based on the CBL2 and the DataMate Application. DataMate is an APP which can be downloaded directly from the CBL2 to the graphing calculator by linking the two devices, setting the calculator to LINK-RECEIVE, then pressing the "TRANSFER" button on the CBL2.

Explore

Posing the Problem:



A Phoenician boat captain was enjoying the cool Mediterranean breeze as his boat sailed from Tyre to Carthage with another shipment of purple dye for the king. The 1200-mile voyage was not easy. The captain smiled to himself as he thought of the many boats from other nations that became lost at sea attempting to make this journey. He and his Phoenician counterparts had a leg up on the competition- they knew how to use the stars to navigate. The captain looked up at the night sky, noticing the stars to make sure he was on course. He was always amazed by the variety of stars, some blue and white, some yellow and red. Some bright, some faint.

We know today that the brightness of a star depends on several factors. One factor is the distance between the Earth and the star. These distances are difficult to measure, so they must be calculated. In order to calculate these distances, astronomers must first know the relationship between the intensity of starlight and the distance between the Earth and the star.

Obtaining and Analyzing the Data:

To solve this problem, we can use the problem-solving strategy of "solving a simpler problem." To do so, use a flashlight to simulate a star and use a light-intensity sensor to measure the intensity of the light for varying distances.

Attach a light sensor to your data collection device and graphing calculator. Run a program, such as the DataMate App, that measures intensity of light to collect data. One person in the group should hold the light sensor as another person walks towards the sensor with the flashlight.

1. **Use the light sensor to collect data in intervals of 0.1 meter. Record your data in the table.**
Participants should determine their own distances. The light sensor (depending on calibration) can detect between 0 and 1 milliwatt per square centimeter. Participants will need to find the first 0.1-meter distance that gives an intensity reading below 1 then collect data from there. The brighter the flashlight, the further away from the flashlight this initial reading will be.

Facilitation Questions
- What part of the light source is the light probe measuring?
The light probe measures the intensity of the light striking it perpendicular to the lens.
- Where should you point the light probe in order to get consistent measurements?
Aim the lens of the light probe toward the center of the flashlight to insure that you are measuring the most direct part of the light beam.

Sample data:

Distance (D) (m)	Intensity (I) (mW/cm^2)
0.6	0.7454
0.7	0.5657
0.8	0.4588
0.9	0.3199
1.0	0.2538
1.1	0.2149
1.2	0.1751
1.3	0.1479
1.4	0.1333
1.5	0.1236
1.6	0.1100
1.7	0.0973
1.8	0.0906
1.9	0.0808
2.0	0.0750

2. **Using an appropriate technology, generate a scatterplot of your data. Sketch your scatterplot.**
Actual scatterplots may vary depending on the technology chosen by the participants.

Facilitation Questions
- What are your independent and dependent variables?
Distance is the independent variable and intensity is the dependent variable.
- What are the domain and range of your data?
Answers may vary. According to the sample data, the domain of the distance is from 0.6 meters to 2.0 meters and the range of the intensity is from 0.0750 to 0.7454 milliwatts per square centimeter.

3. **Find an appropriate function rule to model your data. Test the rule over your scatterplot. Sketch your graph.**
Function rules will vary depending on the data collected. A function rule modeling the sample data is $y = \frac{0.273}{x^2}$.

Facilitation Questions

- What type of function does this data set appear to represent?
The data appear to curve like an inverse variation (rational) function.
- Are the *y*-values increasing or decreasing as the *x*-values increase?
The y-values are decreasing as x increases.
- Is there a constant rate of change?
No.
- What other kinds of parent functions are there in this family?
Any function $y = \frac{k}{x^n}$*, where k and n are constants*
- How can you determine the values of the parameters for that kind of function?
To find the value of k, multiply $x^n y$*. If the value of k is close for all x-y ordered pairs, then the curve is likely to be a good fit.*

4. **A plant will be placed 275 centimeters from the light source. What intensity of light will it receive? Justify your answer.**
Based on the sample data, set up and simplify the equation $y = \frac{0.273}{(2.75)^2} \approx 0.036 \frac{mW}{cm^2}$.

5. **A particular solar cell needs to receive at least 0.4 milliwatts per square centimeter of light to generate enough electricity to power a small toy. How far from the light source should the solar cell be placed in order to begin powering the toy? Justify your answer.**
Based on the sample data, set up and solve the inequality $0.4 \geq \frac{0.273}{x^2}$*. The solar cell should be placed about 0.826 meters, or 82.6 centimeters from the light source.*

Explain

In this phase, use the debrief questions to prompt participant groups to share their responses to the data analysis. At this stage in the professional development, participants should be familiar with using the graphing calculator, a spreadsheet, and TI-Interactive. If none of the participant groups uses one of these three methods, ask them how they could have used that method to analyze the data. This information is important to the discussion of relative advantages and disadvantages of different types of technology. The reasons that a participant group did not choose a particular technology are as important (if not more so) than the justifications a group gives for the technology that they did choose.

1. **How did you develop your scatterplot? Why did you choose this method?**
Ask participants to discuss their methods and their reasons for making that choice. If none of the participants chooses one of the three technologies (graphing calculator, spreadsheet, or TI-Interactive), ask participants why no one made that choice.

See "Technology Tutorial: I've Seen the Light!" for details.

2. **How did you develop your function rule? Why did you choose this method?**
Ask participants to discuss their methods and their reasons for making that choice. If none of the participants chooses one of the three technologies (graphing calculator, spreadsheet, or TI-Interactive), ask participants why no one made that choice.

See "Technology Tutorial: I've Seen the Light!" for details.

3. **How did you solve the problems? Why did you choose this method?**
Ask participants to discuss their methods and their reasons for making that choice. If none of the participants choose one of the three technologies (graphing calculator, spreadsheet, or TI-Interactive), ask participants why no one made that choice.

See "Technology Tutorial: I've Seen the Light!" for details.

Note to Leader: *Record or have a participant volunteer record the responses to Questions 4, 5, and 6 on chart paper to use in the Elaborate phase of the professional development.*

4. **What are the relative advantages and disadvantages of using a graphing calculator to solve this problem?**
Responses may vary.

The data analysis can be done in a few keystrokes. The power to set your own parameters and graph the function rule empowers the participant to use numerical analysis to calculate meaningful parameters such as a constant of variation. The graphical analysis features of the calculator make it easy to use the graph to solve problems by tracing and calculating the intersection of lines.

However, the small screen is difficult to see, and the axes in the window cannot be labeled.

5. **What are the relative advantages and disadvantages of using a spreadsheet to solve this problem?**
Responses may vary.

The regression equation is calculated quickly on the spreadsheet. The axes can be clearly labeled with numbers and text labels. Labeled axes help the participant to use the graph to estimate solutions to problems that can be solved graphically. The graph can be enlarged or *reduced then copied and pasted into other computer documents such as a Word or PowerPoint document to communicate the solution to a problem.*

However, the participant is limited to the regression equations available in the spreadsheet. There are no graphical analysis features in most spreadsheets, so only estimates rather than exact solutions can be obtained graphically.

6. **What are the relative advantages and disadvantages of using TI-Interactive to solve this problem?**
   Responses may vary.

   Like the graphing calculator, data analysis can be done with a few keystrokes and clicks. The function editor enables participants to set their own rational function, empowering them to choose parameters that make physical sense in the context of the problem. The graphical analysis features of TI-Interactive make it easy to use the graph to solve problems by tracing and calculating the intersection of lines.

   Like the spreadsheet, axes can be labeled numerically and with text. The graphs are cleaner and can be copied and pasted into other computer documents.

7. **What TEKS does this activity address?**
   Participants should brainstorm a list of TEKS that they believe they have covered in this activity. The Leader Notes contain a comprehensive list of the TEKS addressed in this phase of the professional development. If participants do not mention some of these TEKS, then ask them how the activity also covers them.

8. **How does the technology that you used enhance the teaching of those TEKS?**
   Responses may vary. However, participants should note that using technology enables them to explore a mathematical concept to a much deeper level. For example, in this activity, using a spreadsheet or the List Editor in a graphing calculator or TI-Interactive allows participants to make quick computations that allow them to determine inverse variation (xy is a constant value) or inverse square variation relationships (x^2y is a constant value).

   Technology makes rich mathematics accessible to a variety of learning styles. For example, students can use a graphing calculator to solve equations and inequalities via tables and graphs rather than merely relying on traditional symbolic manipulation.

I've Seen the Light!: Intentional Use of Data

1. At the close of *I've Seen the Light!*, distribute the **Intentional Use of Data** activity sheet to each participant.

2. Prompt the participants to work in pairs to identify those TEKS that received greatest emphasis during this activity. Prompt the participants to also identify two key questions that were emphasized during this activity. Allow four minutes for discussion.

Facilitation Questions
- Which mathematics TEKS formed the primary focus of this activity?
- What additional math TEKS supported the primary TEKS?
- How do these TEKS translate into guiding questions to facilitate student exploration of the content?
- How do your questions reflect the depth and complexity of the TEKS?
- How do your questions support the use of technology?

3. As a whole group, share responses for two to three minutes.

4. As a whole group, identify the level(s) of rigor (based on Bloom's taxonomy) addressed, the types of data, the setting, and the data sources used during this activity. Allow three minutes for discussion.

Facilitation Question
- What attributes of the activity support the level of rigor that you identified?

5. As a whole group, discuss how this activity might be implemented in other settings. Allow five minutes for discussion.

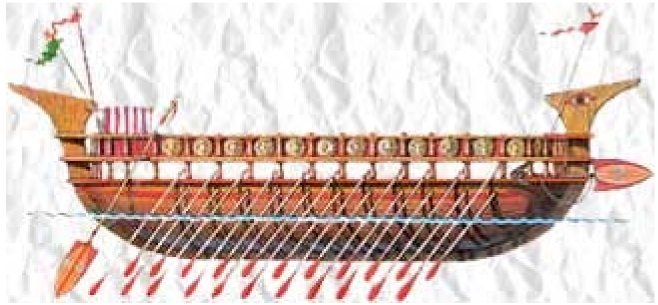
6. Prompt the participants to set aside the completed Intentional Use of Data activity sheet for later discussion. These completed activity sheets will be used during the elaborate phase as prompts for generating attributes of judicious users of technology.

Facilitation Questions
- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) per participant?
- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) per small group of participants?
- How would this activity change if we had access to one computer (one graphing calculator, CBR, etc.) for the entire group of participants?
- How would this activity change if we had used graphing calculators instead of computer-based applications?
- How would this activity change if we had used computer-based applications instead of graphing calculators?
- How might we have made additional use of available technologies during this activity?
- How does technology enhance learning?

Sample Responses:

TEKS		*a(5), a(6), 2A.1B, 2A.10B, 2A.10C, 2A.10D, 2A.10E, 2A.10F, 2A.10G*	
Question(s) to Pose to Students	**Math**	*How did you know what type of function could model the data?* *How did you generate your function rule?*	
	Tech	*How did the technology enable you to collect data?*	
Cognitive Rigor		Knowledge	√
		Understanding	√
		Application	√
		Analysis	√
		Evaluation	√
		Creation	√
Data Source(s)		Real-Time	*The CBL2 generates real-time data*
		Archival	
		Categorical	
		Numerical	
Setting		Computer Lab or CBL2 for each student	*Each student uses the computer or CBL2 to generate their own data.*
		Mini-Lab or CBL2 for each group	*In groups students take turns or groups switch out.*
		One Computer or CBL2 for entire class	*A student operates the control as other students read directions, entire class records data.*
		Graphing Calculator	*Could be used to enter data and find relationships.*
		Measurement-Based Data Collection	*Could be done at stations or individually.*
Bridge to the Classroom		*This activity transfers directly to the classroom with the only modifications being the settings addressed above.*	

I've Seen the Light!



A Phoenician boat captain was enjoying the cool Mediterranean breeze as his boat sailed from Tyre to Carthage with another shipment of purple dye for the king. The 1200-mile voyage was not easy. The captain smiled to himself as he thought of the many boats from other nations that became lost at sea attempting to make this journey. He and his Phoenician counterparts had a leg up on the competition- they knew how to use the stars to navigate. The captain looked up at the night sky, noticing the stars to make sure he was on course. He was always amazed by the variety of stars, some blue and white, some yellow and red. Some bright, some faint.

We know today that the brightness of a star depends on several factors. One factor is the distance between the Earth and the star. These distances are difficult to measure, so they must be calculated. In order to calculate these distances, astronomers must first know the relationship between the intensity of starlight and the distance between the Earth and the star.

To solve this problem, we can use the problem-solving strategy of "solving a simpler problem." To do so, use a flashlight to simulate a star and use a light-intensity sensor to measure the intensity of the light for varying distances.

Attach a light sensor to your data collection device and graphing calculator. Run a program, such as the DataMate APP, that measures intensity of light to collect data. One person in the group should hold the light sensor as another person walks towards the sensor with the flashlight.

1. Use the light sensor to collect data in intervals of 0.1 meter. See ***Technology Tutorial: Using the CBL2 to Collect Light Data*** for detailed instructions if necessary. Record your data in the table.

Distance (D) (m)	**Intensity (I) (mW/cm^2)**

Distance (D) (m)	**Intensity (I) (mW/cm^2)**

2. Using an appropriate technology, generate a scatterplot of your data. Sketch your scatterplot.

3. Find an appropriate function rule to model your data. Test the rule over your scatterplot. Sketch your graph.

4. A plant will be placed 275 centimeters from the light source. What intensity of light will it receive? Justify your answer.

5. A particular solar cell needs to receive at least 0.4 milliwatts per square centimeter of light to generate enough electricity to power a small toy. How far from the light source should the solar cell be placed in order to begin powering the toy? Justify your answer.

I've Seen the Light!: Intentional Use of Data

TEKS			
Question(s) to Pose to Students	Math		
	Tech		
Cognitive Rigor	Knowledge		
	Understanding		
	Application		
	Analysis		
	Evaluation		
	Creation		
Data Source(s)	Real-Time		
	Archival		
	Categorical		
	Numerical		
Setting	Computer Lab		
	Mini-Lab		
	One Computer		
	Graphing Calculator		
	Measurement-Based Data Collection		
Bridge to the Classroom			

Leader Notes: The Doomsday Model

Elaborate

Purpose:

Use a problem context as a catalyst to generate a comparison of the strengths and weaknesses of different technologies. Generate a list of attributes to guide judicious use of technology.

Descriptor:

Participants will use a rational function model for population growth popularly known as the "Doomsday Model," published by three scientists from the University of Illinois in 1960. Participants will obtain actual population data and verify the accuracy of the model using an appropriate technology, then communicate their findings. Participants will revise the model to better fit their data set, if necessary.

Participants will be asked to identify the strengths and weaknesses of using different types of technology. They will generate a list of attributes that can be used to guide judicious use of technology in their classrooms.

Duration:

2 hours

TEKS:

a(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

a(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

2A.1(B) collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

2A.4(A) Identify and sketch graphs of parent functions, including linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$) functions, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x ($f(x) = \frac{1}{x}$).

2A.4(B) Extend parent functions with parameters such as a in $f(x)=\frac{a}{x}$ and describe the effects of the parameter changes on the graph of parent functions.

2A.10(B) analyze various representations of rational functions with respect to problem situations;

2A.10 (C) determine the reasonable domain and range values of rational functions, as well as interpret and determine the reasonableness of solutions to rational equations and inequalities;

2A.10 (D) determine the solutions of rational equations using graphs, tables, and algebraic methods;

2A.10 (E) determine solutions of rational inequalities using graphs and tables;

2A.10 (F) analyze a situation modeled by a rational function, formulate an equation or inequality composed of a linear or quadratic function, and solve the problem; and

TAKS Objectives Addressed by these Algebra 2 TEKS:

- Objective 1: Functional Relationships
- Objective 2: Properties and Attributes of Functions
- Objective 10: Mathematical Processes and Mathematical Tools

Technology:

- Internet access
- Graphing Calculator
- TI-Interactive
- Spreadsheet
- Graph linking capability, such as TI-Connect or Casio Program-Link

Materials:

Advanced Preparation: Transparencies

Presenter Materials: projector (computer or overhead) for graphing calculator

Per group: Internet access, sentence strips

Per participant: graphing calculator, activity sheets

Leader Notes:

In this phase of the professional development, participants will solve a problem by gathering data from the Internet, using their choice of technology to analyze the data, and be able to justify their choices. This activity will frame a discussion in which participants will be asked to identify the strengths and weaknesses of using different types of technology. They will generate a list of attributes that can be used to guide judicious use of technology in their classrooms.

Posing the Problem:

In 1960, Heinz von Foerster, Patricia Mora, and Larry Amiot, three scientists from the University of Illinois, published "Doomsday: Friday, 13 November, AD 2026" in the journal *Science*. In their paper, they considered the past population growth of the world and the current state (as of 1960) of the world's resources and their ability to sustain a certain population. They developed a model to describe population growth. A simplified variation of this model, where *t* represents the year and *P* represents the world population in billions, is:

$$P = \frac{195}{2026 - t}$$

They used this rational function to decide when the world's population would reach an unsustainable level and called this date "doomsday."

Use the Internet to obtain world population data since 1960. How well did the Doomsday Model describe the world's population growth between 1960 and 2000? How well does the model describe the world's population today? Based on the population data you found, how would you revise the model? When does this model predict "doomsday" will occur?

Share your results and your revised model with the group.

Facilitation Questions

- What kind of function is the Doomsday Model? What are its attributes?
  Answers may vary.
- What kind of function appears to model the actual population data? How do you know?
  Answers may vary.
- Which representation of the data would be most helpful?
  Answers may vary. Some may feel a scatterplot would be more helpful, and others would prefer a tabular or symbolic approach.
- Which technology would enable you to build this representation the most efficiently?
  Answers may vary, depending on the comfort level and experience of the participants with Excel, TI-Interactive, or the graphing calculator.

Leader Note: there are many possible solutions to this problem. In this phase of the professional development, it is more important to probe participants' reasoning for making their choices of technology. Participants' reasoning will help them build a framework for choosing the most appropriate technology in their day-to-day classroom instruction at the end of this phase of the institute.

One possible solution:

Participants can obtain world population data from the United Nations, the Central Intelligence Agency's World Factbook, or the U.S. Census Bureau. They can also obtain the data via online almanacs such as www.infoplease.com.

According to the United States Census Bureau:

Address http://www.census.gov/ipc/www/worldpop.html

U.S. Census Bureau

Total Midyear Population for the World: 1950-2050

Year	Population	Average annual growth rate (%)	Average annual population change
1950	2,556,517,137	1.47	37,798,160
1951	2,594,315,297	1.61	42,072,962
1952	2,636,388,259	1.71	45,350,197
1953	2,681,738,456	1.77	47,979,452
1954	2,729,717,908	1.87	51,465,740
1955	2,781,183,648	1.89	52,974,870
1956	2,834,158,518	1.95	55,842,882
1957	2,890,001,400	1.94	56,522,767
1958	2,946,524,167	1.76	52,351,768
1959	2,998,875,935	1.39	42,090,531
1960	3,040,966,466	1.33	40,782,196
1961	3,081,748,662	1.80	55,995,030
1962	3,137,743,692	2.19	69,519,033
1963	3,207,262,725	2.19	71,119,386
1964	3,278,382,111	2.08	68,979,816
1965	3,347,361,927	2.07	70,182,601
1966	3,417,544,528	2.02	69,689,877
1967	3,487,234,405	2.04	71,794,577
1968	3,559,028,982	2.07	74,579,864
1969	3,633,608,846	2.05	75,142,514

According to www.infoplease.com:

Year	*Total world population (mid-year figures)*	*Ten-year growth rate (%)*
1950	*2,556,000,053*	*18.9%*
1960	*3,039,451,023*	*22.0*
1970	*3,706,618,163*	*20.2*
1980	*4,453,831,714*	*18.5*
1990	*5,278,639,789*	*15.2*
2000	*6,082,966,429*	*12.6*

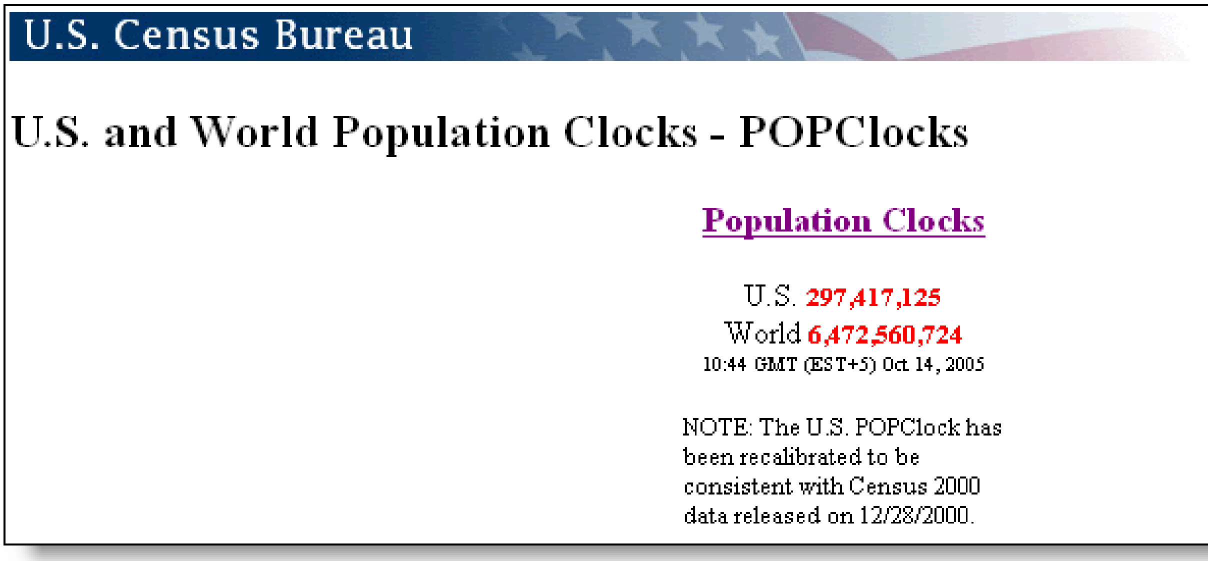
Use the function editor of a graphing calculator to build a table of the Doomsday Model function:

X	Y1	
1985	4.7561	
1990	5.4167	
1995	6.2903	
2000	7.5	
2005	9.2857	
2010	12.188	
2015	17.727	

X=2000

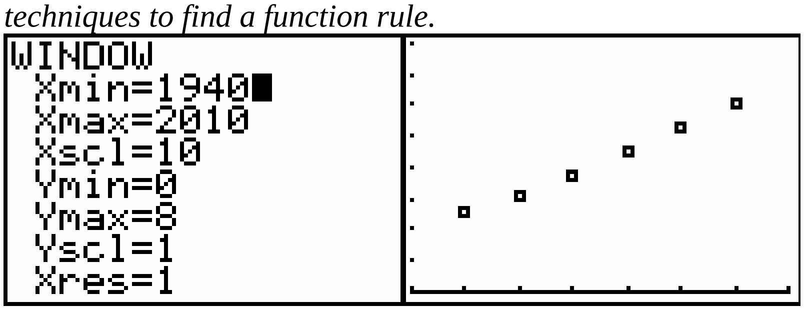
For the year 2000, the model predicted a population of about 7.5 billion people, and the actual population was about 6.08 billion, so the model actually overestimated the world's population.

According to the U.S. Census Bureau, www.census.gov,



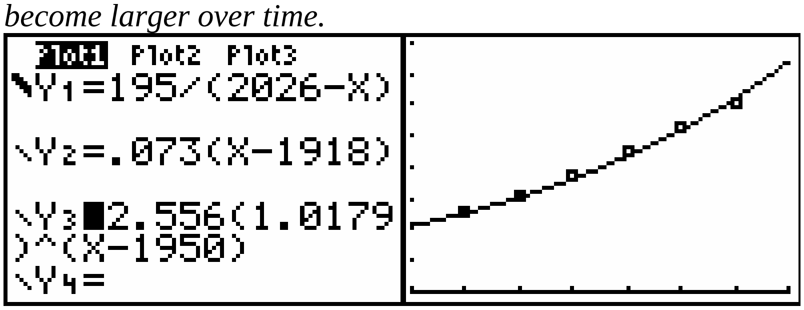
For today (actual year might vary, sample data shown for 2005), the model predicted a population of about 9.29 billion people, but the actual population estimate is only about 6.47 billion. Again, the model has overestimated the world's population.

To revise the model, use actual population data. Generate a scatterplot, then use curve-fitting techniques to find a function rule.

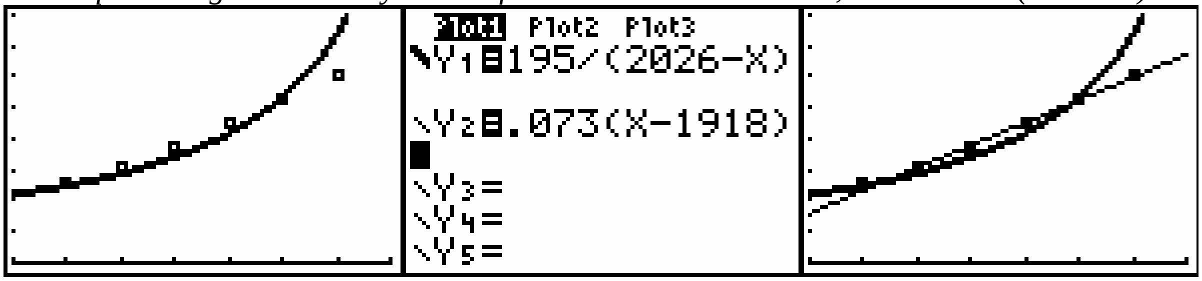


The Doomsday Model fits the data well until about 1990. However, the 2000 data point is well below the Doomsday Model curve (bold).

Possible Revised Models Include:
Population growth tends to be exponential. Try using transformations to generate an exponential function that will model the data. If participants use an exponential function, then there is no "doomsday" asymptote, but the population will continue to increase and the increase will become larger over time.



Visual inspection of the data reveals that the data appear to be linear. Rounded values of the least-squares regression line yield the function P = 0.073x – 140, or P = 0.073(x – 1918).



In factored form, the linear function has an x-intercept of 1918, meaning that in the year 1918, the world population was 0. Obviously, the model is not valid for years prior to 1950. In terms of determining the year of "doomsday," a linear function has no asymptotes. According to a linear model, the population can increase infinitely at a constant rate.

Debriefing the Activity:

1. *Upon completion of the technology-based activity, prompt participants to work in pairs to brainstorm the role(s) technology played in this activity.*

2. *Repost the Venn diagram summaries from the Engage phase.*

3. *Prompt participants to collect the "green sheets" from each Explore/Explain phase, the summaries about the intentional use of data that followed each Explore/Explain phase.*

4. *Display the **Transparency: Teaching Strategies** and prompt participants to reflect on the following question, "How do the summaries on the Venn diagrams, our summaries about the use of data, and the activities reflect the following four teaching strategies for developing judicious users of technology?"*

Facilitation Questions

- How have the experiences in this professional development promoted careful decision-making about the appropriate use of technology?
  Answers may vary.
  E/E 1 example: technology can make complex problems accessible to all students
  E/E 2 example: comparing graphing calculator and spreadsheet to make scatterplots and generate function rules
  E/E 3 example: technology expands possible sources of data that can be used to explore functional relationships
  Elaborate example: there are multiple sources of Internet data, so the source of the data must be carefully considered
- How was technology used as a tool for the teaching and the learning of the TEKS?
  Answers may vary.
  E/E 1 example: use of calculator to solve a problem with complicated arithmetic
  E/E 2 example: the use of Geometer's Sketchpad to collect data
  E/E 3 example: use of light probe and CBL2 to collect data
  Elaborate example: use of the Internet to collect data to test and verify a model

- When was technology use promoted? Why?
  Answers may vary.
  E/E 1 example: participants are prompted to use a graphing calculator to generate a scatterplot and function rule
  E/E 2 example: participants are prompted to use technology to generate function rules, but are not told which technology to use
- When was technology use restricted? Why?
  Answers may vary. Overall, the use of technology was not overtly restricted in the TMT3 Algebra 2 module. However, NCTM suggests in their 2005 Yearbook, Technology-Supported Mathematics Learning Environments, that restricting the use of technology is an appropriate way to encourage learners to more judiciously choose which technologies to use in problem-solving and when to use them.
- How did the technology support anticipatory, or "what if…", thinking about "algebraic insight"?
  Answers may vary. Sample answers might include:
  Technology empowers students to quickly use transformations for curve-fitting to a set of data, building algebraic insight into functional relationships.
  Technology makes complex data sets and data collections accessible to all students.

5. *Post **Transparency 1: Looks Like – Sounds Like**. Prompt the participants to respond to the following statement and question: "A successful teacher is one who uses technology judiciously. What does this ideal teacher look like and sound like?" as described on Transparency 1: Looks Like – Sounds Like. Record the participant responses on sentence strips. Post the sentence strips randomly so that they are visible to the entire group. Use participants as scribes as needed to facilitate the recording process.*

6. *Post **Transparency 2: Looks Like – Sounds Like**. Prompt the participants to respond to the following statement and question: "A successful student is one who uses technology judiciously. What does this ideal student look like and sound like?" as described on Transparency 2: Looks Like – Sounds Like. Record the participant responses on sentence strips. Post the sentence strips randomly so that they are visible to the entire group. Use participants as scribes as needed to facilitate the recording process.*

7. *Direct the participants to work in small groups to brainstorm categories for classifying the "looks like" and "sounds like" responses.*

Facilitation Questions

- Do any of these responses require the teacher or the student to make decisions about technology use? Is this important? Should we add some responses?
  Answers may vary.
- Do any of these responses reflect decision making about how to best integrate technology? Is this important? Should we add some responses?
  Answers may vary.
- Do any of these responses reflect decision making about when to use or when not to use technology? Is this important? Should we add some responses?
  Answers may vary.
- Do any of these responses reflect the need for thinking about how the technology provides "algebraic insight"? Is this important? Should we add some responses?
  Answers may vary.

8. *As a whole group, debrief the categories created by small groups. Reorganize the sentence strips into broad categories. As a whole group, create titles for each of these categories. Record each title on a separate sheet of chart paper. Post the chart paper and reorganize the related sentence strips as shown below. Enlist participants to help with this process.*

Sample Category: Student Choice

The teacher allows students to select the computer or the graphing calculator and refrains from commenting while students decide.

The student chooses to use a scatterplot instead of a table to represent her data.

9. *Prompt the participants to consider adding additional statements to any of the categories listed above that are not already posted. Reorganize "looks like, sounds like" sentence strips as needed.*

10. *Distribute to each group sentence strips that are a different color than the previously used sentence strips. Prompt each group to generate two classroom suggestions for each* ***category****. Examples may include "Students monitor their own use and misuse of technology," "Include examples that require technology use," or "Do not allow students to use technology until after predictions are made and justified."*

11. Prompt participants to post their sentence strips as shown below.

Sample
Category:
Student Choice

The teacher allows students to select the computer or the graphing calculator and refrains from commenting while students decide.

The teacher provides a card whose front and back sides are two different colors, one color corresponding to calculator, one to computer. Students can display their choice of technology by placing the card with one color face up.

The teacher and students brainstorm a "pros and cons" chart to develop for the computer and the graphing calculator and then prompts students to select a tool.

12. Ask the participants to summarize any trends or patterns observed in the classroom suggestions.

13. Read the statement by Ball and Stacey found on ***Transparency: Student Research*** *as a closing thought to this phase of the professional development.*

Facilitation Question

- What is the value of this statement?
  Answers may vary. It is encouraging to read that technology use is teachable. It makes me consider how I might better meet the needs of the student who doesn't struggle with the math yet struggles with the technology.

Transparency: Teaching Strategies

"How do the summaries on the Venn diagrams, our summaries about the use of data, and the activities reflect the following four teaching strategies for developing judicious users of technology?"

Judicious users of technology:

a. Promote careful decision-making about the appropriate use of technology.

b. Integrate technology whenever relevant to the mathematical learning goals.

c. Promote and restricts the use of technology when appropriate for promoting mathematical learning

d. Promote anticipatory thinking about "statistical insight," "algebraic insight," or "geometric insight."

Transparency 1: Looks Like – Sounds Like

A successful **teacher** is one who uses technology judiciously.

What does this ideal **teacher** look like and sound like in this activity?

Looks like…	Sounds like…

Transparency 2: Looks Like – Sounds Like

A successful **student** is one who uses technology judiciously.

What does this ideal **student** look like and sound like during the completion of this activity?

Looks like…	Sounds like…

Transparency: Student Research

Research by Pierce (2002) indicates that some students are always judicious users and others persist with passive or random, unthinking use. However, she found that a large, middle group can be helped to learn to work judiciously.

Ball & Stacey, 2005, p. 5

Ball, L., & Stacey, K. (2005). Teaching strategies for developing judicious technology use. In Masalski, W. J., & Elliott, P. C. (Eds.), *Technology-supported mathematics learning environments, sixty-seventh yearbook*, pp. 3-16. Reston, VA: National Council of Teachers of Mathematics.

The Doomsday Model

In 1960, Heinz von Foerster, Patricia Mora, and Larry Amiot, three scientists from the University of Illinois, published "Doomsday: Friday, 13 November, AD 2026" in the journal *Science*. In their paper, they considered the past population growth of the world and the current state (as of 1960) of the world's resources and their ability to sustain a certain population. They developed a model to describe population growth. A simplified variation of this model, where t represents the year and P represents the world population in billions, is:

$$P = \frac{195}{2026 - t}$$

They used this rational function to decide when the world's population would reach an unsustainable level and called this date "doomsday."

Use the Internet to obtain world population data since 1960. How well did the Doomsday Model describe the world's population growth between 1960 and 2000? How well does the model describe the world's population today? Based on the population data you found, how would you revise the model? When does this model predict "doomsday" will occur?

Share your results and your revised model with the group.

Evaluate

Purpose:
Evaluate judicious uses of technology in the mathematics classroom.

Descriptor:
Participants will review the instructional phases of this professional development and the classroom-ready lessons according to the list of attributes generated in the elaborate phase of the professional development. Participants may make revisions to the list of attributes. Participants will engage in discussion about how each lesson exhibits a judicious use of technology; participants will address the question, "How does the use of technology in this student lesson help me teach the concepts and skills more effectively and efficiently?"

Duration:
2 hours

Materials:
- Small (1" x 1.5") restickable notes
- Chart paper
- Markers
- Tape to adhere chart paper to the wall

Leader Notes:
The Evaluate phase is a time for participants to reflect upon their experiences and apply their knowledge to a new situation. The facilitator can deduce from the participants' actions how well they have been able to develop a sense of the judicious use of technology, including when it is appropriate or not appropriate to use technology to teach the mathematics TEKS. Further, participants should be able to discern when it is appropriate to use which technology.

Use the following steps to conduct the Evaluate phase of the institute.

1. *Distribute small restickable notes to each participant.*
2. *Assign different phases of this professional development to pairs of participants.*
3. *Prompt each pair of participants to use the restickable notes to highlight locations in each phase of the professional development that make judicious use of technology, according to the criteria on the* ***Transparency: Encouraging Judicious Use of Technology****. The restickable notes should be used to highlight those attributes of the teaching strategies outlined during the Elaborate Phase of this professional development.*
4. *After each pair has had time to evaluate the given phase of the professional development, prompt each pair of participants to create a summary of its findings on chart paper.*

   Sample responses might include:

Using the Internet to gather real-time data to test mathematical models makes the mathematics more relevant to students.

Data collection via technology allows students to focus on the concept of functional relationships instead of getting bogged down in non-technology data collection.

Using technology to collect data saves valuable time in the classroom. Instead of spending a whole class period generating data with paper and pencil constructions, students can generate the same data set in minutes.

Using Excel to generate scatterplots and regression equations allows students to build graphics that can be pasted into Word or PowerPoint quickly.

5. *Identify a location in the room for each phase of the professional development. Direct participants to post their summaries in the appropriate location.*
6. *Perform a gallery walk through each phase, asking participants to determine which teaching strategies for judicious use of technology seemed to have the greatest impact on the given phase.*
7. *Prompt participants to share any new thoughts that should be added to the classroom suggestions for each teaching strategy.*
8. *Distribute the classroom-ready lessons to each participant. Prompt each participant to continue the evaluation process for judicious use of technology, using the classroom-ready lessons as the context for evaluation. The participants should use the restickable notes to highlight those parts of each lesson that reflect the four teaching strategies for developing judicious use of technology.*
9. *As time allows, offer small-group and whole-group opportunities for participants to share what participants highlighted.*
10. *Redirect participants' attention to the four statements made at the beginning of the professional development session. Ask the participants if they would "shift" the placement of their sticky dots. If they respond with a "Yes," ask the participants why they would shift the placement of their sticky dots.*
11. *Draw an end to the professional development session with a parting thought rather than a closing thought so that participants leaving thinking, "How will I use what I learned?" rather than, "That was a good session." Examples of such parting thoughts include:*
    a. *As you leave, please consider ways that you might include the use of data and technology in your classroom next week.*
    b. *As you leave, please consider how you might best make use of the computer or computers available for your classroom use.*
    c. *As you leave, please consider how students might be equipped to ask better questions about what they are learning when they have graphing calculators in their hands.*

Transparency: Encouraging Judicious Use of Technology

- How did the activity promote careful decision making about the use of technology?

- How did the activity integrate technology into the learning of mathematics?

- Was technology use ever restricted for the purpose of enhancing learning? Why?

- How did the technology facilitate discussion about "algebraic sense"?

Gallery Walk Observations

Explore/Explain I: Flying Off the Handle	How did the activity promote careful decision making about the use of technology? How did the activity integrate technology into the learning of mathematics? Was technology use ever restricted for the purpose of enhancing learning? Why? How did the technology facilitate discussion about "algebraic sense"?

Explore/Explain II: A Golden Idea	How did the activity promote careful decision making about the use of technology? How did the activity integrate technology into the learning of mathematics? Was technology use ever restricted for the purpose of enhancing learning? Why? How did the technology facilitate discussion about “statistical sense”, “algebraic sense”, or “geometric sense”?

Explore/Explain III: I've Seen the Light!	How did the activity promote careful decision making about the use of technology? How did the activity integrate technology into the learning of mathematics? Was technology use ever restricted for the purpose of enhancing learning? Why? How did the technology facilitate discussion about "statistical sense" or "algebraic sense"?

Elaborate: The Doomsday Model	How did the activity promote careful decision making about the use of technology? How did the activity integrate technology into the learning of mathematics? Was technology use ever restricted for the purpose of enhancing learning? Why? How did the technology facilitate discussion about “statistical sense” or “algebraic sense”?

2A.1A	The student is expected to identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations.
2A.1B	The student is expected to collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.
2A.3A	The student is expected to analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems.
2A.3B	The student is expected to use algebraic methods, graphs, tables, or matrices, to solve systems of equations or inequalities.
2A.3C	The student is expected to interpret and determine the reasonableness of solutions to systems of equations or inequalities for given contexts.

Materials

Advanced Preparation:

- Student/teacher access to computers with TI Interactive and/or a graphing calculator and a projection device to use TI Interactive as a class demonstration tool.
- Chart paper
- Markers
- Student and group copies of handouts

For each student:

- Graphing calculator
- Student worksheets, *Casey's Part – Time Jobs, Selected Response Questions*
- Access to TI Interactive and TI Interactive Video Rental Sketches 1, 2, 3, and Video Rental Spreadsheet 1

For each student group of 3 to 4 students:

- Chart paper
- Markers
- Group worksheets *Andy and Beca, Video Joe, Shipping Costs*
- Access to TI Interactive and TI Interactive Video Rental Sketches 1, 2, 3, and Video Rental Spreadsheet 1

ENGAGE

The Engage portion of the lesson is designed to create student interest in the topic of video rentals, which will be explored later in the lesson as linear programming. This part of the lesson is designed for groups of three to four students.

1. Have students in groups of 3 or 4.
2. Distribute one copy of *Andy and Beca* to each group of students.
3. Working together, each group should complete the worksheet.
4. Each group will prepare a poster on their responses to Questions 12 – 14.
5. Post the chart papers around the room. Each group should choose one person from the group to stay with the group poster and explain their responses. Have the groups conduct a gallery walk to see all the other posters and solution strategies.
6. Come back together as a large group and debrief. Possible facilitation questions are shown below.

Facilitation Questions – Engage Phase

- How were the posters/solutions the same?
  Answers may vary.
- How were the posters/solutions different?
  Answers may vary.
- What were some of the strategies used?
  Answers may vary. Some groups may have solved the problem graphically, some may have used a table containing a list of the possible outcomes, some may have used logical reasoning to solve the problems.
- Did you see any posters/solutions that included graphs? How were they used?
  Answers may vary. If a student group used a graph to solve the problem, they likely have a shaded region bounded by the lines $x = 5$, $y = 7$, and $x + y = 10$.
- Did you see any posters/solutions that included tables? How were they used?
  Answers may vary. If a student group used a table, they may have a 3-column table: number of VHS tapes, number of DVDs, and total cost.

EXPLORE

The Explore portion of the lesson provides the student with an opportunity to be actively involved in the exploration of the mathematical concepts addressed. This part of the lesson is designed for students to work in groups of 3 to 4 students.

1. Distribute copies of *Video Joe* to each group of students.
2. Have groups work through *Video Joe*.
3. When finished, each group should record their solutions on chart paper, answering all the questions included in the situation.
4. Have each group post their chart paper.

Facilitation Questions – Explore Phase

- How does the amount of shelf space limit the number of DVDs and VHS tapes Joe can stock?
  He can only stock 1500 inches worth of videos.
- What other information in the problem limits the number of DVDs and VHS tapes he can stock?
  Twice as many VHS as DVD, between 80 and 200 VHS
- What happened to the region as more inequalities, or restrictions, were applied?
  The region became more limited.
- What does this limited region mean in the situation?
  Each restriction in the situation limits the number of VHS tapes and DVDs that can be stocked.
- Do all the charts have the same feasible regions shown? How are they the same or different?
  Graphs could have different windows/domain and range settings or different scaling.

EXPLAIN

The Explain portion of the lesson is directed by the teacher to allow the students to formalize their understanding of the TEKS addressed in the lesson.

1. Debrief the *Video Joe* activity. Use group posters when asking the Facilitation Questions.

Facilitation Questions – Explain Phase

- What is the inequality representing the number of VHS tapes and DVDs Video Joe can stock on his shelves? $5x + 4y \leq 1500$ Why is the inequality less than 1500 and not greater than 1500? *Video Joe can stock fewer videos, but not more videos that take up 1500 inches of shelf space.* What do the '5' and '4' represent in the situation? *each VHS tape takes up 4 inches of shelf space and each DVD takes up 5 inches of shelf space*
- Could you have graphed the inequality in standard form, by hand? *Yes, by using the x-intercepts and y-intercepts.* What do the *x*-intercept and *y*-intercept represent in the situation? *The x-intercept represents the number of DVDs Video Joe could stock if he had no VHS tapes and the y-intercept represents the number of VHS tapes Video Joe could stock if he had no DVDs.*
- What is the inequality in slope-intercept form? $y = -\frac{5}{4}x + 375$ Why is it important to put the inequality in slope-intercept form? *to plot the inequality quickly into TI Interactive or a graphing calculator.* What does the *y*-intercept represent in the situation? *the y-intercept represents the number of VHS tapes Video Joe could stock if he had no DVDs.*

Facilitation Questions – Explain Phase, continued

- What window settings did you use when graphing the inequality? Why? *Answers will vary. They should range from 0 – 400, approximately. It is important for them to understand why negative values are not necessary in this situation.*
- How did the inequality $y \le 2x$ restrict the possible combinations of VHS tapes and DVDs Video Joe could stock? *Of the region represented by the first inequality, now the only combinations that can be used are the ones where the number of VHS tapes is at least twice the number of DVDs.*
- How did limiting the number of VHS tapes (between 80 and 200) restrict the possible combinations of VHS tapes and DVDs Video Joe could stock? *The number of possible combinations were decreased and limited to y-values between 80 and 200.*
- What happened to the original region as restrictions were added to the situation? *The original region got smaller each time a restriction was added.*

- The area common to all the restrictions of the situation is called the feasible region. Is the point (50 , 250) inside the feasible region? *No* Explain your answer in terms of the situation. *This point exceeds 200 VHS tapes, so therefore does not fit within the restriction that the number of VHS tapes has to be between 80 and 200.* Is the point (150 , 100) inside the feasible region? *Yes* Explain your answer in terms of the situation. *It meets all the restrictions of the situation.*
- Where is the point (100 , 200) in regards to the feasible region? *At the intersection of two of the inequalities.* How is this point different from the other two points you have looked at? *This point is located at an intersection of two inequalities, and therefore lies on the edge of the feasible region.* This point is one of the vertices of the feasible region. What are the coordinates of the other vertices? *(140 , 200), (236 , 80), (40 , 80).* What do all the vertices points have in common? *They all occur at the intersection of two of the restrictions of the feasible regions.* What are some methods that were used on the graphing calculator or in TI Interactive for finding the coordinates of the vertices? *Table, trace, calculate the intersection, graph*
- When calculating the amount of profit Video Joe could make, are there other combinations of VHS tapes and DVDs that would generate a different amount of profit (other than those listed in the table)? *Yes* Do you think there are any of those combinations that would generate more than $660 or less than $240 in profit? *No* Why or why not? Use the spreadsheet to verify your answer. *Students should enter various points in the spreadsheet trying to generate more than $660 or less than $240. By using the spreadsheet portion of TI Interactive, the students can readily see the profit being generated by the different combinations of VHS tapes and DVDs. The vertices of the feasible region contain the extremes of the region, therefore from those points you will find the maximum/minimum profit for Video Joe's store.*

ELABORATE

The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS within a new situation. This part of the lesson is designed for students working in small groups.

1. Distribute **Shipping Costs** to each group of students. Divide students into an even number of groups.
2. All groups should have access to TI Interactive or a graphing calculator.
2. Have groups work together and record their actions on chart paper. Students should include all aspects of solving the problem on the chart paper and include sketches of the feasible region, inequalities, and justification of their solution using the cost function.
3. Put two groups together and have them explain to each other how they arrived at their solutions. After both groups have explained their strategies, the two groups should decide what their two posters have in common and how are they different.
4. Each pair of groups will present a summary to the whole class of the various strategies used, what their two posters had in common and how they were different. Use the Facilitation Questions to debrief the activity.

Facilitation Questions – Elaborate Phase

- How is this situation different from *Video Joe*?
  This is asking for a minimum cost vs. maximum profit in Video Joe
- What were some of the things all the posters had in common?
  Answers may vary.
- What were some of the differences between the posters?
  Answers may vary.
- Describe the restrictions or limitations in this situation.
  Truck capacity, capacity of each case, minimums/maximums of types of videos needed
- What were the window settings for the graphs? Why were these values chosen?
  Answers may vary. Window settings should be chosen that are appropriate to the domain and range of the situation.
- Did you have to adjust the window settings of your graph as you added restrictions to the graph? Why?
  Answers may vary. Depending on the original window chosen, sometimes adding restrictions will force students to zoom in on a certain region or to enlarge their window to see new vertex points.

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.

1. Distribute *Casey's Part-time Jobs* to each student.
2. Each student should complete the assessment, showing all appropriate work.
3. Upon completion of the activity sheet, a rubric should be used to assess student understanding of the concepts addressed in the lesson.

Answers and Error Analysis for selected response questions:

Question Number	*TEKS*	*Correct Answer*	*Conceptual Error*	*Conceptual Error*	*Procedural Error*	*Procedural Error*	*Guess*
1	2A.1A	B	A	C	D		
2	2A.3A	A	C	D	B		
3	2A.3C	C	A	B			D
4	2A.3B	B	C	D	A		

Andy and Beca

Andy and Beca are renting videos for the weekend. They can only to afford to rent a maximum of six videos. Some of the videos must be on VHS tapes and some must be on DVD.

1. What are the possible combinations of VHS and DVD Andy and Beca can rent? Use the table to list all the possible combinations.

VHS	DVD
1	*1*
1	*2*
1	*3*
1	*4*
1	*5*
2	*1*
2	*2*
2	*3*
2	*4*
3	*1*
3	*2*
3	*3*
4	*1*
4	*2*
5	*1*

Open **Video Rental Sketch 1** through TI Interactive.

2. What does each plotted point represent on the graph?
   Each point represents one possible combination of renting VHS tapes and DVDs.

3. Why are there no points with negative coordinates plotted on the graph?
   You cannot rent a negative amount of tapes of DVDs.

4. Does your table match the table shown on the sketch?
   Answers may vary.

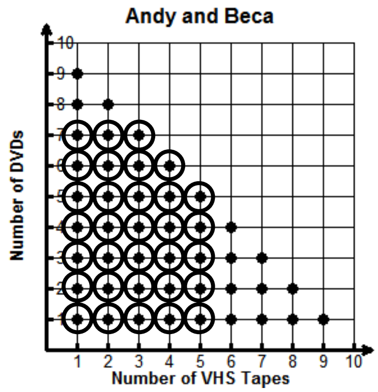
5. Predict how the graph would change if Andy and Beca could rent a total of 10 videos.
   There would be more combinations possible, therefore more points on the graph.

6. Open **Video Rental Sketch 2** to check your prediction. How do the two graphs compare? Explain.
   Answers may vary.

7. What are the total number of combinations of renting VHS tapes and DVDs?
   45 combinations

8. If Andy and Beca limit the number of VHS tapes to 5 or less and the number of DVDs to 7 or less, how would the graph change?
There would be 13 fewer points, or combinations of VHS tapes and DVDs.

9. Shade the graph below to show the new restrictions to the number of VHS tapes and DVDs Andy and Beca could rent.



10. What are the possible combinations, with the new restrictions included?
32 possible combinations
11. What are the outermost points of the restricted region?
(1 , 7) (3 , 7) (5 , 5) (5 , 1) (1 , 1)

12. If VHS tapes rent for \$4 and DVDs rent for \$2, what is the most they could spend if they stay within all the restrictions? What combination of VHS tapes and DVDs would be?
\$30 for 5 VHS tapes and 5 DVDs.

13. If VHS tapes rent for \$3 and DVDs rent for \$4, what is the most they could spend? What combination of VHS tapes and DVDs would that be?
\$37 for 3 VHS tapes and 7 DVDs.

14. How did the cost change from the first situation to the second situation? Why?
An increase of \$7; VHS tapes cost less and DVDs cost more in the second situation.

Video Joe

Video Joe has decided to open a small video rental store. He plans on offering DVDs and VHS tapes for rental. After installing all the shelves in the store, he calculates that he has 125 feet of shelf space to store the DVDs and VHS tapes. Each DVD takes up 5 inches of shelf space, while each VHS tape takes up 4 inches of shelf space.

Let x = the number of DVDs and y = the number of VHS tapes he can stock on his shelves at any given time.

1. Write an equation describing the number of VHS tapes and DVDs Video Joe can stock on his shelves, given the limited amount of shelf space.
   $5x + 4y = 1500$ *125 feet = 1500 inches*

2. Would Video Joe be able to stock more or less VHS tapes and DVDs than represented by the equation? Justify your answer.
   Less. With a defined amount of shelf space, Video Joe can always put fewer videos on the shelves, but it is impossible to put more.

3. Write the equation as an inequality to represent this situation.
   $5x + 4y \le 1500$

4. Write the inequality in slope – intercept form.
   $5x + 4y \le 1500$
   $4y \le -5x + 1500$
   $y \le -\frac{5}{4}x + 375$

5. Graph this inequality on TI Interactive or graphing calculator. Describe the region that would apply to this inequality.
   The x-intercept is (300 , 0) and the y-intercept is (0, 375). The region under the line applies to this inequality since Video Joe can use less than 1500 inches of shelf space, but not more.

6. Video Joe would like to keep at most 2 times as many VHS tapes as DVDs. Write an inequality to represent this restriction.
   $y \le 2x$

7. Graph this inequality on TI Interactive or graphing calculator on the same screen as the previous inequality. Describe the region that now applies to the two restrictions (inequalities).
   The common area that is located beneath each of the lines.

8. Video Joe would also like to keep between 80 and 200 VHS tapes in stock. Write two inequalities to represent this restriction.

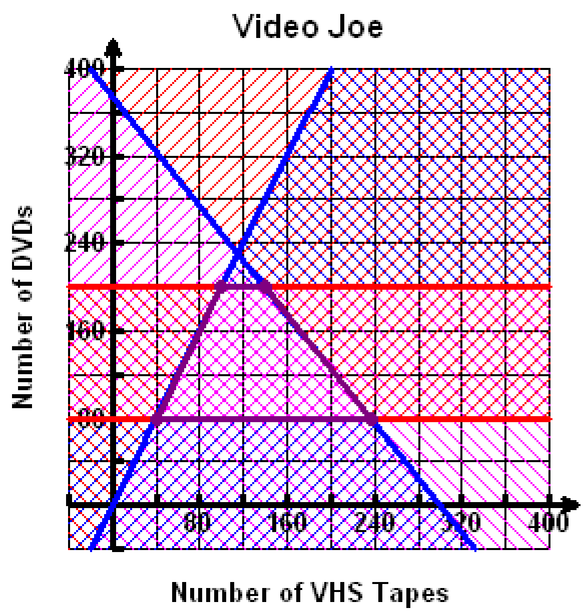
   $y \geq 80$
   $y \leq 200$ or $80 \leq y \leq 200$

9. Graph these two inequalities on TI Interactive or graphing calculator on the same screen as the two previous inequalities. What region represents all the restrictions, or inequalities, in this situation?

   The common area beneath the first two inequalities and between $y = 80$ and $y = 200$.

10. Open **Video Rental Sketch 3**. How does your graph compare to this one? Explain. (The purple trapezoidal region represents the region common to all restrictions.)

    Answers may vary.



11. What are the vertices of the region common to all the restrictions (**feasible region**)?

    (100 , 200) (140 , 200) (236 , 80) (40 , 80)

12. What do these coordinates represent in this situation?

    100 DVDs, 200 VHS tapes
    140 DVDs, 200 VHS tapes
    236 DVDs, 80 VHS tapes
    40 DVDs, 80 VHS tapes

13. Video Joe makes a profit of $2.25 on each DVD rented and $1.50 on each VHS tape rented. Write a function representing the profit he makes if he rents x number of DVDs and y number of VHS tapes.

$f(x,y) = 2.25x + 1.50y$

14. Use the profit function to determine the amount of profit Video Joe would make using the coordinates of the feasible region.
Students could use a calculator or the spreadsheet feature on TI Interactive to calculate each value.

15. Use the spreadsheet in TI Interactive to enter the coordinates of the feasible region and the profit function. Open **Video Rental Spreadsheet 1** to verify your answers. How do these answers compare with yours? Explain any differences.
Answers may vary.

DVDs	VHS	Profit
100	200	600
140	200	660
236	80	534
40	80	240

16. Which combination would generate the most profit for Video Joe, but still meet all the restrictions? How do you know?
140 DVDs and 200 VHS tapes will generate $660 for Video Joe. Using a trial-and-error approach, students will not find another combination of DVDs and VHS tapes that will generate profit greater than $660.

Shipping Costs

Video Joe orders all his DVDs and VHS tapes from an area supplier. The supplier has only one truck available for delivery and it has a capacity of 3600 cubic feet. One case of VHS tapes takes up 18 cubic feet of space, while one case of DVDs takes up 12 cubic feet of space. Video Joe places an order with the supplier for one truckload of VHS tapes and DVDs. He has to order between 150 and 240 cases of DVDs to meet the demand and at least 20 cases of VHS tapes. The shipping costs are based on the number of cases on the truck. Each case of VHS tapes costs $3.50 in shipping costs and each case of DVDs costs $3.75 in shipping costs.

Let x = number of cases of VHS tapes
y = number of cases of DVDs

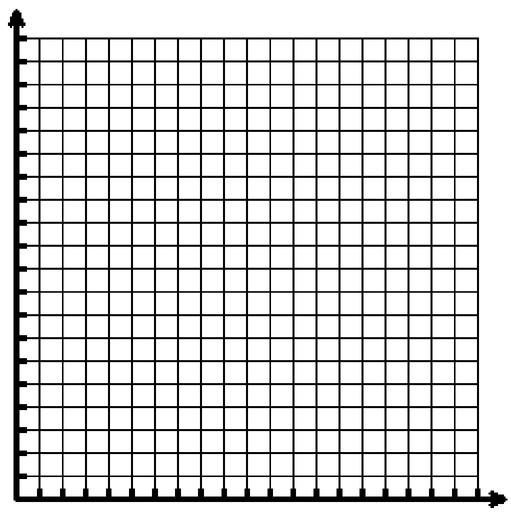
How many cases of VHS tapes and DVDs should he order if he would like to pay the least amount possible in shipping costs and stay within all the restrictions?
20 cases of VHS tapes and 150 cases of DVDs

Casey's Part-time Jobs

Casey the college student is working two part-time jobs. He works at a video rental store for $5.25 per hour and at a movie theatre for $6.05 per hour. He wants to work no more than 30 hours per week. He wants to work between two and three times the hours at the movie theatre than at the video store. He also has to work a minimum of 10 hours per week at the movie theatre.

Let x = the number of hours worked at the video store
y = the number of hours worked at the movie theatre

Use TI-Interactive or a graphing calculator to graph the feasible region described above. Record the feasible region below. Label the axes and the vertices of the feasible region.



How many hours should he work at each job to earn the maximum amount of money each week?
What is the maximum amount of money he could make each week? Justify your answers.
He should work 10 hours a week at the video store and 20 hours a week at the movie theatre.
$173.50

Andy and Beca

Andy and Beca are renting videos for the weekend. They can only to afford to rent a maximum of six videos. Some of the videos must be on VHS tapes and some must be on DVD.

1. What are the possible combinations of VHS and DVD Andy and Beca can rent? Use the table to list all the possible combinations.

VHS	DVD

Open **Video Rental Sketch 1** through TI Interactive.

2. What does each plotted point represent on the graph?

3. Why are there no points with negative coordinates plotted on the graph?

4. Does your table match the table shown on the sketch?

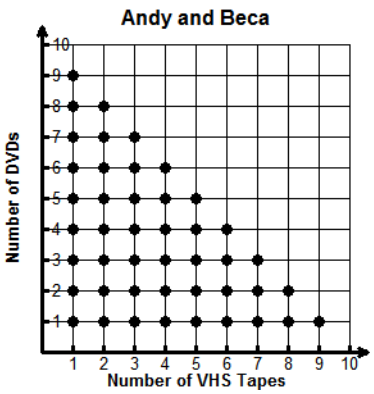
5. Predict how the graph would change if Andy and Beca could rent a total of 10 videos.

6. Open **Video Rental Sketch 2** to check your prediction. How do the two graphs compare? Explain.

7. What are the total number of combinations of renting VHS tapes and DVDs?

8. If Andy and Beca limit the number of VHS tapes to 5 or less and the number of DVDs to 7 or less, how would the graph change?

9. Shade the graph below to show the new restrictions to the number of VHS tapes and DVDs Andy and Beca could rent.



10. What are the possible combinations, with the new restrictions included?

11. What are the outermost points of the restricted region?

12. If VHS tapes rent for $4 and DVDs rent for $2, what is the most they could spend if they stay within all the restrictions? What combination of VHS tapes and DVDs would that be?

13. If VHS tapes rent for $3 and DVDs rent for $4, what is the most they could spend? What combination of VHS tapes and DVDs would that be?

14. How did the cost change from the first situation to the second situation? Why?

Video Joe

Video Joe has decided to open a small video rental store. He plans on offering DVDs and VHS tapes for rental. After installing all the shelves in the store, he calculates that he has 125 feet of shelf space to store the DVDs and VHS tapes. Each DVD takes up 5 inches of shelf space, while each VHS tape takes up 4 inches of shelf space.

Let x = the number of DVDs and y = the number of VHS tapes he can stock on his shelves at any given time.

1. Write an equation describing the number of VHS tapes and DVDs Video Joe can stock on his shelves, given the limited amount of shelf space.

2. Would Video Joe be able to stock more or less VHS tapes and DVDs than represented by the equation? Justify your answer.

3. Write the equation as an inequality to represent this situation.

4. Write the inequality in slope – intercept form.

5. Graph this inequality on TI Interactive or graphing calculator. Describe the region that would apply to this inequality.

6. Video Joe would like to keep at most 2 times as many VHS tapes as DVDs. Write an inequality to represent this restriction.

7. Graph this inequality on TI Interactive or graphing calculator on the same screen as the previous inequality. Describe the region that now applies to the two restrictions (inequalities).

8. Video Joe would also like to keep between 80 and 200 VHS tapes in stock. Write two inequalities to represent this restriction.

9. Graph these two inequalities on TI Interactive or graphing calculator on the same screen as the two previous inequalities. What region represents all the restrictions, or inequalities, in this situation?

10. Open **Video Rental Sketch 3**. How does your graph compare to this one? Explain. (The purple trapezoidal region represents the region common to all restrictions.)

11. What are the vertices of the region common to all the restrictions (**feasible region**)?

12. What do these coordinates represent in this situation?

13. Video Joe makes a profit of $2.25 on each DVD rented and $1.50 on each VHS tape rented. Write a function representing the profit he makes if he rents x number of DVDs and y number of VHS tapes.

14. Use the profit function to determine the amount of profit Video Joe would make using the coordinates of the feasible region.

15. Use the spreadsheet in TI Interactive to enter the coordinates of the feasible region and the profit function. Open **Video Rental Spreadsheet 1** to verify your answers. How do these answers compare with yours? Explain any differences.

16. Which combination would generate the most profit for Video Joe, but still meet all the restrictions? How do you know?

Shipping Costs

Video Joe orders all his DVDs and VHS tapes from an area supplier. The supplier has only one truck available for delivery and it has a capacity of 3600 cubic feet. One case of VHS tapes takes up 18 cubic feet of space, while one case of DVDs takes up 12 cubic feet of space. Video Joe places an order with the supplier for one truckload of VHS tapes and DVDs. He has to order between 150 and 240 cases of DVDs to meet the demand and at least 20 cases of VHS tapes. The shipping costs are based on the number of cases on the truck. Each case of VHS tapes costs $3.50 in shipping costs and each case of DVDs costs $3.75 in shipping costs.

Let x = number of cases of VHS tapes
y = number of cases of DVDs

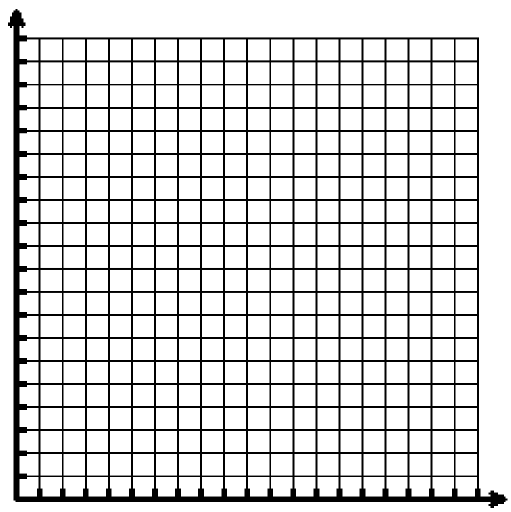
How many cases of VHS tapes and DVDs should he order if he would like to pay the least amount possible in shipping costs and stay within all the restrictions?

Casey's Part-time Jobs

Casey the college student is working two part-time jobs. He works at a video rental store for \$5.25 per hour and at a movie theatre for \$6.05 per hour. He wants to work no more than 30 hours per week. He wants to work between two and three times the hours at the movie theatre than at the video store. He also has to work a minimum of 10 hours per week at the movie theatre.

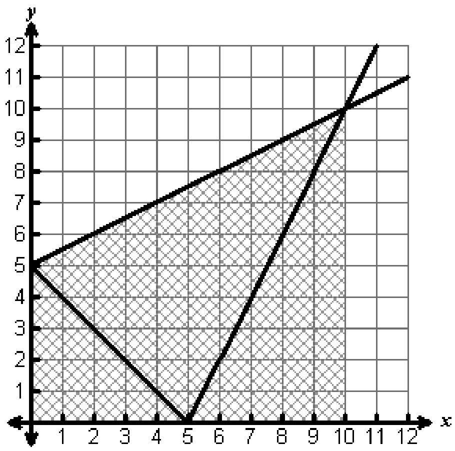
Let x = the number of hours worked at the video store
y = the number of hours worked at the movie theatre

Use TI-Interactive or a graphing calculator to graph the feasible region described above. Record the feasible region below. Label the axes and the vertices of the feasible region.



How many hours should he work at each job to earn the maximum amount of money each week? What is the maximum amount of money he could make each week? Justify your answers.

1 Shown below is a feasible region. The profit function for the region is $f(x, y) = 6x + 5y$.



What are the minimum and maximum values of the function?

A 5 and 10

B 25 and 110

C 25 and 30

D 0 and 110

2 A company machines and sells nuts and bolts. One machine can produce 100 nuts and 50 bolts each hour. Due to demand, the company must produce at least two times more nuts than bolts. The machine runs a maximum of 40 hours per week. Which set of inequalities shows the correct restrictions for this situation if x represents the number of nuts produced in one week and y represents the number of bolts produced in one week?

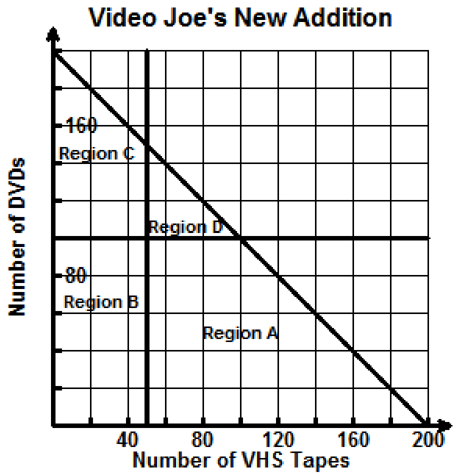
A $\frac{x}{100} + \frac{y}{50} \le 40$
$y \ge \frac{1}{2}x$

B $\frac{x}{50} + \frac{y}{100} \le 40$
$y \ge \frac{1}{2}x$

C $\frac{x}{100} + \frac{y}{50} \le 40$
$y \ge 2x$

D $\frac{x}{50} + \frac{y}{100} \le 40$
$y \ge 2x$

3 Video Joe is expanding his video store. He added enough shelving to hold a maximum of 200 items. He wants to have 50 VHS tapes at most and at least 100 DVDs in stock at all times in the new addition.



Which of the following regions represents the limited restrictions of this situation?

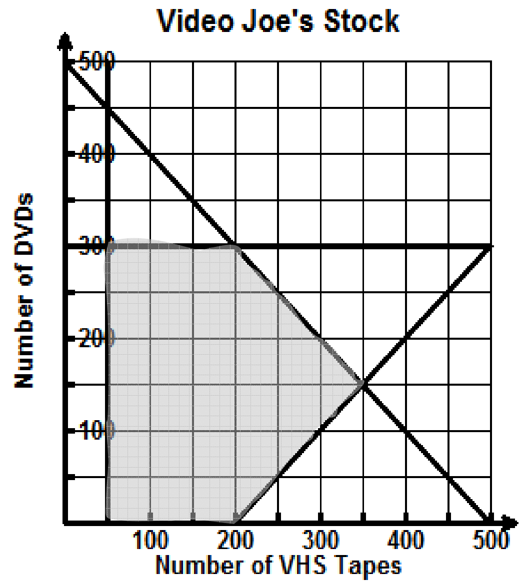
A Region A

B Region B

C Region C

D Region D

4 The feasible region shown below represents the possible amounts of VHS tapes and DVDs on Video Joe's shelves at any given time.



If he makes $1.75 on each VHS rental and $2.00 on each DVD rental, which combination of VHS tapes and DVD rentals would result in the most profit?

A 350 VHS and 150 DVD

B 200 VHS and 300 DVD

C 150 VHS and 350 DVD

D 300 VHS and 200 DVD

2A.1(B)	collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.
2A.4(B)	extend parent functions with parameters such as *a* in $f(x) = a/x$ and describe the effects of the parameter changes on the graph of parent functions
2A.8(B)	analyze and interpret the solutions of quadratic equations using discriminants and solve quadratic equations using the quadratic formula
2A.11(B)	use the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describe limitations on the domains and ranges, and examine asymptotic behavior
2A.11(C)	determine the reasonable domain and range values of exponential and logarithmic functions, as well as interpret and determine the reasonableness of solutions to exponential and logarithmic equations and inequalities
2A.11(F)	analyze a situation modeled by an exponential function, formulate an equation or inequality, and solve the problem.
G.1(B)	recognize the historical development of geometric systems and know mathematics is developed for a variety of purposes
G.2(A)	use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships
G.5(A)	use numeric and geometric patterns to develop algebraic expressions representing geometric properties
G.5(B)	use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles;
G.11(A)	use and extend similarity properties and transformations to explore and justify conjectures about geometric figures
G.11(B)	use ratios to solve problems involving similar figures

Materials

Advanced Preparation:

- Student access to computers with Geometer's Sketchpad and necessary sketches and/or a projection device to use the computer activities as a demonstration
- Student or teacher access to the Internet
- Chart paper and markers or blank transparencies
- Graphing calculator connected to an overhead projector or presenter
- Transparencies provided with this lesson
- Copies of "Technology Tutorial: The Golden Ratio" for each group of students.

For each student:
- Graphing calculator
- **The Eye of the Beholder** activity sheet
- **Creating a "Golden" Exponential Function** activity sheet
- **Algebra and the Golden Ratio** activity sheet
- **The Golden Ratio in Art and Architecture** activity sheet
- **Golden Areas** activity sheet

ENGAGE

The Engage portion of the lesson is designed to create student interest in the application of the number phi (golden ratio) to everyday life. This part of the lesson is designed for groups of two to four students using a computer station for Internet access and The Geometer's Sketchpad.

1. Show **Transparency – The Eye of the Beholder**.
2. Hand each student a copy of the activity sheet **The Eye of the Beholder** and have each student group open the Geometer's Sketchpad sketch ***Face Sample***. Students will use Geometer's Sketchpad to obtain data to calculate ratios of the facial features indicated on the sketch. They will record these ratios on the activity sheet. Each ratio will be approximately 1.6.
3. Groups will also log on to the Internet and use the website shown on their activity sheet or similar sites that have photos of celebrities. Each group is to find one celebrity photo that they want to copy and insert into Geometer's Sketchpad. If your school's firewall blocks the use of such sites, you can download some photos yourself and give them to the students in a Word file. They can then copy and insert whichever photo they choose.
4. Students may want to open a new sketch in which to insert their celebrity photo instead of inserting into the same file as the sample. Once students have inserted a copy of the celebrity photo, they will use Geometer's Sketchpad and take the same types of measurements as on the face sample and record these on the activity sheet.
5. Once the groups have recorded their measurements and ratio values, have them share their findings with the whole class. You may want to record their findings on chart paper or a transparency so students can have fun discussing who is the most handsome or beautiful.

Facilitation Questions – Engage Phase

- What facial features do you find attractive in another person?
  Answers may vary. Let students briefly share to enhance the engagement aspect.
- Besides ratios, what other mathematical terms or concepts can you apply to a person's physical appearance?
  Symmetry, orientation of features (crooked smile, etc.), overall size of head compared to body
- Do certain feature measurements of species other than humans have an affect on their "attractiveness"?
  Animals look for features in their potential mates such as overall proportion, strength, speed, etc. to ensure that their offspring have a greater chance of survival. Judges at dog and cat shows look for certain physical features to judge the pureness of a breed.
- What are some of the different ways that ratios can be expressed?
  Ratios can be written as fractions or decimals or using the word "to" or a colon to separate the terms of the ratio.
- What is the decimal value (to the nearest tenth) of the ratios found from the face sample?
  They all were close to 1.6.
- Consider the following ratios. What are their decimal equivalents and what do you notice? $\frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}$
  Each new numerator is equal to the sum of the numerator and denominator from the previous fraction; the decimal values seem to approach about 1.6.
- How do the decimal equivalents in the sequence of ratios above compare to the ratios in your celebrity's facial feature ratios?
  In the most attractive faces, the ratios were close to 1.6 – same as the ratios in the sequence.

EXPLORE

The Explore portion of the lesson provides an opportunity for the student to connect the concept of the golden ratio to an exponential function. This part of the lesson is designed for groups of two to four students working with Geometer's Sketchpad and a graphing calculator.

1. Distribute the **Creating a "Golden" Exponential Function** activity sheet.
2. Inform students that there are several geometric models of the golden ratio. A representation of the exact value of the golden ratio will be discovered through patterns of golden triangles. Display **Transparency 1 – The Golden Triangle**. On the transparency, sketch a bisector of $\angle BAC$ and name its intersection with the leg $\overline{BC}$ point D.

3. Prompt students to recognize characteristics of $\triangle ABC$ and $\triangle CAD$ that they may remember from Geometry such as:

Facilitation Questions – Explore Phase

- What are the measures of the base angles of $\triangle ABC$? Why?
  72°. If the vertex angle is 36°, that leaves 180-36 = 144 degrees to split equally among the two base angles.
- If you bisect $\angle BAC$, what are the measures of the two new angles? Why?
  36°, because bisecting an angle cuts the angle measure in half.
- How are $\triangle ABC$ and $\triangle CAD$ related?
  They are both isosceles triangles with 36° vertex angles.
- Are $\triangle ABC$ and $\triangle CAD$ similar? How do you know?
  Yes, because all corresponding angles are congruent.
- What does this imply about the lengths of certain sides of $\triangle ABC$ and $\triangle CAD$? Why?
  Corresponding sides have proportional lengths, such as $\frac{BC}{AC} = \frac{AD}{DC}$. *Other proportions can be made using corresponding sides in a similar fashion.*

4. Students should open the Geometer's Sketchpad sketch **Golden Triangle** and find the measurements and ratios indicated. To find the measurements, students can click on the appropriate action buttons.
5. Prompt students to enter their measurements on the activity sheet. Each group will probably have different segment measurements if they resized the triangle in the sketch. However, the ratios should all be the same, about 1.6.
6. Use **Transparency 2 – The Golden Triangle** to show how to create more triangles within the original triangle. As more triangles are created, help students see each new golden ratio. Use colored markers if possible. The **Golden Triangle 2** page in the Geometer's Sketchpad sketch **Golden Triangle** has the triangles already created, but students will have to find the measurementss on their own.
7. Students could spend lots of time listing every ratio from the seven nested triangles. However, our emphasis now is to show how the golden triangles are connected to an exponential function using 1.6 as the common ratio (e.g., "b" in the function $y = a \bullet b^x$.) The value of "a" would be whatever the student's initial leg length is. Do NOT encourage them to use the exponential regression. If you do, make sure you have a discussion about how the values are related to the data.
8. Important! Have students share the function that they derived and how they calculated it.

Facilitation Questions – Explore Phase

- What happens if you change the size of your triangle in either of the sketches?
  The side lengths will change but the ratios all remain about 1.618.
- How many proportions can you make from the segments in **Golden Triangle 2**?
  Answers may vary but there's a lot.
- Why are you asked to write an exponential function instead of a linear, quadratic, or other type of function?
  From y-value to y-value, there is a common ratio which is about 1.618.
- What window did you use to display your data? Why?
  The window depends on the size of the largest segment recorded, $\overline{BC}$ *. A possible window could be x: 0 , 10, 1 and y: 0, 10, 1.*
- How did you use your table to develop an exponential function for your data?
  Answers may vary. Students may have found successive ratios of leg lengths in the table. They may have guessed at the initial value by using transformations of the graph.
- Is your function exactly the same as other students' functions? Why or why not?
  No, the "b" value is the same (1.618) but the "a" value may be different. If the original triangle is changed in size, the side lengths will vary. However, all of the triangles are "golden," so the ratio of leg to base will always equal phi and is the ratio of the exponential function.
- How can you use the table, graph, and/or function to find the next term in the sequence of leg lengths?
  Table: multiply the preceding term by 1.618
  Graph: trace on the function curve to x = 8 and read the y-value
  Function: on the home screen, enter x = 8 into the function rule and evaluate
- Where would the 8th term value appear on the set of golden triangles?
  Extend side $\overline{AC}$ *out to the left. Construct a 36°* *angle with vertex B so that one side contains* $\overline{BA}$ *and the other side intersects* $\overline{AC}$ *. Label this point of intersection point P. The length of* $\overline{BP}$ *is the 8th term. Its value should equal BC ÷ 1.618.*
- If you created another triangle inside of $\triangle GHC$, describe the side that fits the data in your table.
  Bisect $\angle GHC$ *and name its intersection with* $\overline{GC}$ *point J. The length of side* $\overline{HJ}$ *has a value approximately equal to GC ÷ 1.618. This value would fit before the first term and equals the initial value used in the function.*

EXPLAIN

The Explain portion of the lesson is directed by the teacher to allow the students to formalize their understanding of the actual value of the golden ratio, known as phi.

1. Refer to **Transparency – The Golden Section** to connect the golden ratio to the Fibonacci sequence. Revisit the proportion of the golden ratio on **Transparency – The Algebra of the Golden Ratio** in order to derive the exact value of the ratio.
2. Lead students through the discussion of solving the proportion. You may want to stay with the variable "a" when solving, then introduce the symbol for *phi* at the end.
3. Give students time to solve the quadratic equation and assist as needed. They should be able to come up with the exact value $\frac{1+\sqrt{5}}{2}$ or at least a good decimal approximation of about 1.61803.

   Sample solution using the quadratic formula:

$$\Phi^2 = \Phi + 1$$

$$\Phi^2 - \Phi - 1 = 0$$

$$\text{Let } a = 1,\ b = -1,\ c = -1$$

$$\Phi = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$\Phi = \frac{1 \pm \sqrt{1+4}}{2}$$

$$\Phi = \frac{1 \pm \sqrt{5}}{2}$$

$\frac{1-\sqrt{5}}{2} < 0$*, so it is an extraneous solution and* $\Phi = \frac{1+\sqrt{5}}{2}$.

4. Students will connect the Fibonacci sequence to another exponential function using the activity sheet, **Algebra and the Golden Ratio**. Give students time to work on the activity sheet with a partner then discuss their observations and results.

Facilitation Questions – Explain Phase

- Is the value of the golden ratio really a fraction/ratio?
  *Technically, a fraction or rational number is a ratio of whole numbers so the "golden ratio" is **not** a fraction.*
- Are the ratios made from consecutive Fibonacci numbers the same as the golden ratio?
  These ratios are approximations. The larger the Fibonacci numbers used, the better the approximation to phi. (Actually the limit of these ratios = the golden ratio but that may be a discussion for another class!)
- How did you solve the quadratic equation?
  Answers may vary. Students could use the quadratic formula or find a reasonable approximation on a graphing calculator.
- Is there just one value for the golden ratio?
  Yes and no. When you solve the quadratic, you get two answers, but the larger value of 1.61803… is commonly accepted as the value of the golden ratio, often called φ. Curiously however, the other value, 0.61803… shares the same decimal part and is equal to $\frac{1}{\varphi}$.

ELABORATE

The Elaborate portion of the lesson provides the student with an opportunity to extend what they've learned to real-word applications in art and architecture. This part of the lesson is designed for students to work in groups of two to four.

1. Show how the numbers in the Fibonacci sequence approximate phi in the golden rectangle model. See **Transparency – The Golden Rectangle**.
2. Students will search the Internet for "golden ratio" and "art" or "architecture."
3. Give each group a copy of the activity sheet **The Golden Ratio in Art and Architecture**.
4. Students are to find one example of how the golden ratio has been used in architecture and one example of art (painting, sculpture, etc.). Each group will record their findings on the activity sheet and present their findings to the class.

Facilitation Questions – Elaborate Phase

- What examples did you find of the golden ratio?
  Answers will vary but will probably include structures such as the pyramids, the Parthenon, the United Nations building, and the Notre Dame cathedral. Art may include works by Leonardo da Vinci, Georges Seurat, Rembrandt, and Salvador Dali.

Facilitation Questions – Elaborate Phase

- What are some other names for the golden ratio?
  Golden mean, golden section, phi, tau (uncommon), divine proportion

- Did you run across any other examples of the golden ratio?
  Answers may vary. In nature, spirals in flower petals, seed heads, pine cones, and leaves on stalks come in Fibonacci numbers. Shells such as the nautilus shell is formed in a spiral that illustrates the golden ratio.

- Did you come across any other models of the golden ratio other than the golden rectangle?
  Probably so. Many websites show the golden ratio as it relates to a pentagon, a pentagram, a decagon, and a golden triangle.

- Did you find any symbols or notations that were new to you?
  The symbol for phi, φ, is foreign to most students and is the only one we want to address in this lesson.

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson. This assessment is intended for groups of two to four students.

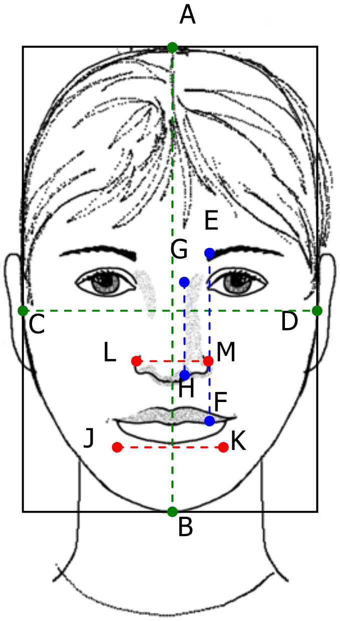
1. Provide each group a copy of the activity sheet **Golden Areas**.
2. Provide each student with a graphing calculator.
3. Upon completion of the activity sheet, a rubric should be used to assess student understanding of the concepts addressed in the lesson.

Answers and Error Analysis for selected response questions:

Question Number	*TEKS*	*Correct Answer*	*Conceptual Error*	*Conceptual Error*	*Procedural Error*	*Procedural Error*	*Guess*
1	2A.11F	A	B	D			C
2	2A.11B	B	A		D		C
3	2A.8B	C	A	B	D		
4	2A.1B	C	B		D		A

The Eye of the Beholder – *Answer Key*

1. Study the features on the artist's sketch below. Identify the segments that represent each of the following ratios.



Sketch by artist, Debra L. Hayden, 2005.
Used by permission

Ratio	Segments
Length of face, to Width of face	$\frac{\text{Length of face}}{\text{Width of face}} = \frac{AB}{CD} = 1.63$
Lips to eyebrows to Length of nose	$\frac{\text{Lips to eyebrows}}{\text{Length of nose}} = \frac{EF}{GH} = 1.61$
Width of mouth to Width of nose	$\frac{\text{Width of mouth}}{\text{Width of nose}} = \frac{JK}{LM} = 1.60$
Average Ratio	1.61

2. Open the sketch **Face Sample** in Geometer's Sketchpad. Calculate the ratio values indicated in the sketch by clicking on the "Measure Ratio" action button for the given ratio. Also, calculate the average ratio. Record your answers in your table.

3. Log on to the Internet and open the website http://www.angelfire.com/celeb2/celebrityfaces/. Search for a photo of your favorite celebrity. The photo must be a full front view of the face.

4. Right click on the face and select "Copy" so that you can "insert" the photo into Geometer's Sketchpad.

5. Using Geometer's Sketchpad, construct and measure segments of the face you copied as shown on the sample. Measure the appropriate ratios and record them in the chart below. See "Technology Tutorial: The Golden Ratio" for assistance with the technology.

I used a photo of : *Brad Pitt*

Length of face	*5.02 cm*	Ratio	*1.65*
Width of face	*3.05 cm*		
Lips to eyebrows	*1.77 cm*	Ratio	*1.62*
Length of nose	*1.09 cm*		
Width of mouth	*1.20 cm*	Ratio	*1.39*
Width of nose	*0.86 cm*		

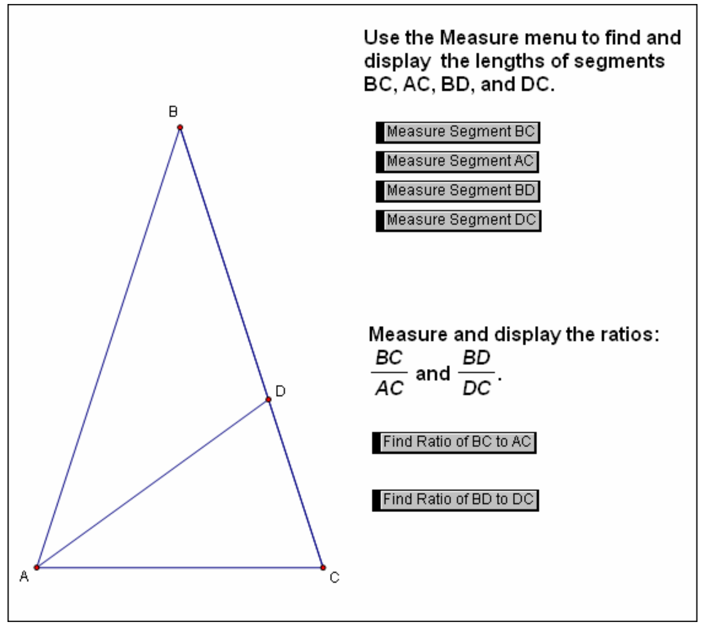
Answers will vary depending on photo chosen. However, ratios will probably be between 1.4 and 1.8.

6. How do your ratios compare with those found by other groups in the class? Why do you think this is so?
   Answers may vary. Ratios should all be similar and fairly close to 1.61.

Creating a "Golden" Exponential Function

Answer Key

1. Open the sketch **golden triangle1** to find possible measurements for each of the following:



Answers will vary.

The length of $\overline{BC}$ = *9.6776*

The length of $\overline{AC}$ = *5.9811*

The ratio of $\overline{BC} : \overline{AC} \approx$ *1.62*

Answers will vary.

The length of $\overline{BD}$ = *5.9811*

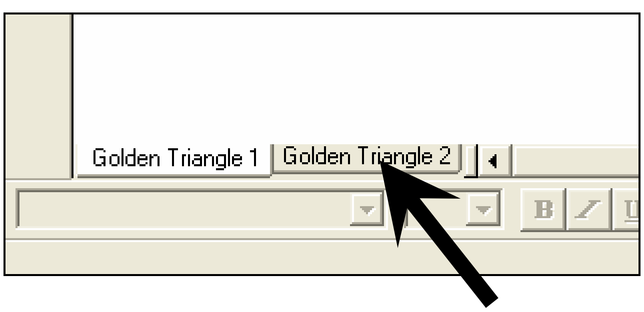
The length of $\overline{DC}$ = *3.6965*

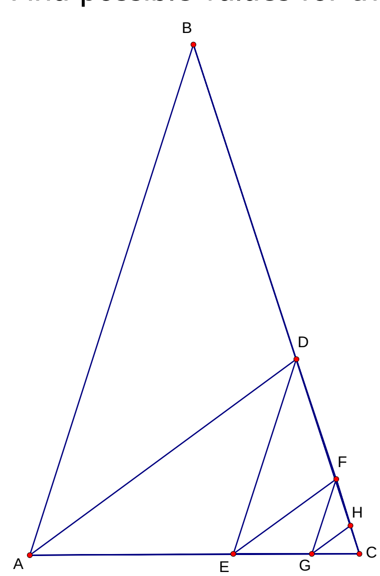
The ratio of $\overline{BD} : \overline{DC} \approx$ *1.62*

2. Click on Point *C* and drag it around the screen. What happens to the segment lengths?
As the triangle gets bigger, the lengths increase. As the triangle gets smaller, the lengths decrease.

3. What happens to the ratios when you drag point *C* around the screen?
The ratios stay the same, no matter how big the triangle gets.

4. Click the **Golden Triangle 2** tab inside your sketch. Find possible values for the following:

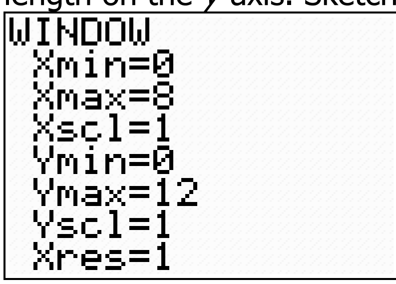


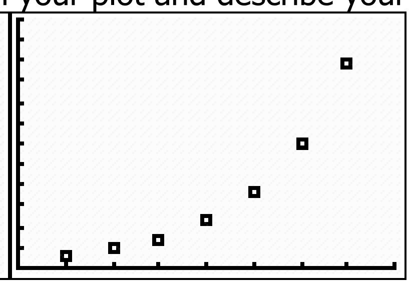


Triangle	Leg	Length	Successive Ratios
1	$\overline{HC}$	*0.546*	
2	$\overline{GC}$	*0.883*	*1.62*
3	$\overline{FC}$	*1.429*	*1.62*
4	$\overline{EC}$	*2.312*	*1.62*
5	$\overline{DC}$	*3.741*	*1.62*
6	$\overline{AC}$	*6.053*	*1.62*
7	$\overline{BC}$	*9.794*	*1.62*

Answers will vary

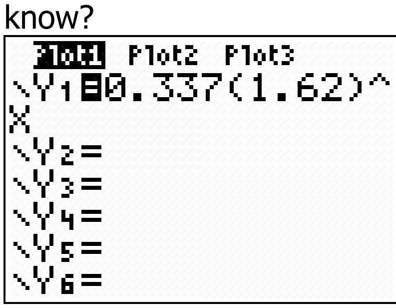
5. Enter the numbers 1 – 7 into List 1 on your graphing calculator. Enter the lengths of the segments $\overline{HC}$, $\overline{GC}$, $\overline{FC}$, $\overline{EC}$, $\overline{DC}$, $\overline{AC}$, and $\overline{BC}$ into List 2. Create a scatter plot on your graphing calculator with the Triangle number on the *x*-axis and the Leg length on the *y*-axis. Sketch your plot and describe your window.

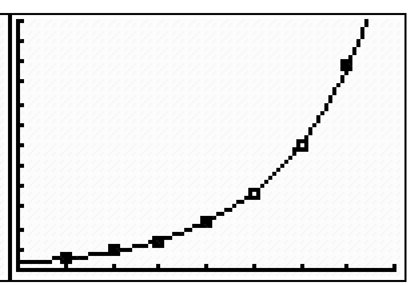




6. Determine an exponential function that passes through these points. Explain how you determined the function.
Answers may vary. By finding the successive ratios, the exponential function $y = 0.337\left(1.62\right)^x$ *can be generated.*

7. Sketch your plot and function graph. Does the function fit the data well? How do you know?





8. What does the coefficient in your function represent in the golden triangle? How did you obtain this value?
The coefficient represents the leg length of the "0" triangle, or the one preceding triangle GHC in the sequence. In other words, the coefficient is the initial value when x = 0. I had to divide the first leg length by 1.62 to find the y-value that corresponds with x = 0.

9. What does the base of the power in your function represent in the golden triangle?
The base represents the successive ratio between consecutive terms. In this case, it is the golden ratio rounded to approximately 1.62.

Algebra and the Golden Ratio

Answer Key

You have found the exact value of the golden ratio to be $\frac{1+\sqrt{5}}{2}$. Let's look at how this value connects to the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, …

Consider the table below but don't fill in the right-hand column until you've answered questions 1 – 3.

Term number	Fibonacci number
	5
1	*8*
2	*13*
3	*21*
4	*34*
5	*55*
6	*89*
7	*144*
8	*233*
9	*377*
10	*610*

1. If you made a scatter plot of Fibonacci number vs. term number, what would the scatter plot look like?
   It would look like an exponential curve except the first few points are off.

2. If you started with 1 as your first Fibonacci number, could you write a function that would pass through all of the points in your scatter plot?
   Starting with y=1 means that the first few points don't fit very well.

3. How could you make a scatter plot that more closely fits an exponential function?
   Shift to about the 4th or 5th Fibonacci number. I started with 8 and the graph was easier to fit.

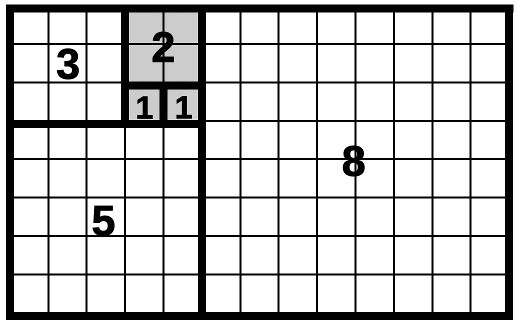
4. Fill in the table with the Fibonacci numbers of your choice and write an exponential function to fit your points.
   $y = 5 * 1.6^x$

5. Which would give a better fit: starting with 5 or starting with 13? How does choosing a different starting number affect your function rule?
   The further down the Fibonacci sequence you go, the closer the ratios of consecutive terms are to 1.6, so starting with 13 would be better than if you start with 5.

Golden Areas

Answer Key

Consider the squares that make up a golden rectangle shown below. The squares have sides that are terms of the Fibonacci sequence: 1, 1, 2, 3, 5,… Each golden rectangle, such as the square that is shaded, is formed by attaching the next Fibonacci square to the previous golden rectangle.



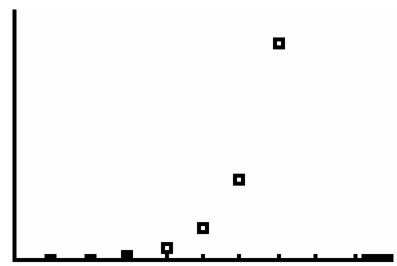
1. Complete the table below to show the relationship between the number of a square in each golden rectangle and the area of the square. Let the 3 × 3 square be square #1.

Square Number	Area of Square
1	*9*
2	*25*
3	*64*
4	*169*
5	*441*
6	*1156*
7	3025

2. Enter the square numbers into List 1 of a graphing calculator and the areas into List 2. Make a scatter plot for squares 1 – 7. Sketch your scatter plot below and describe the domain and range of the plot.

   Domain {1, 2, 3, …7}

   Range is between 9 and 3025



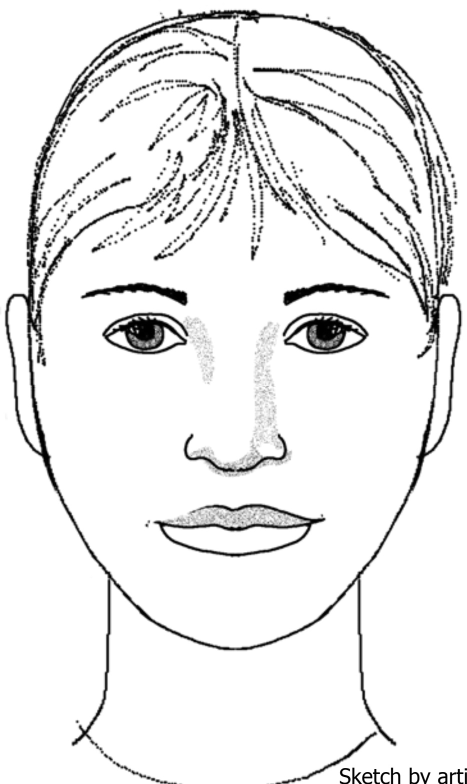
3. Without using the regression feature on your calculator, write a function that fits your data. Enter your function into your calculator to test it. Alter the function as needed until you are satisfied that it fits the data.
   $y = 4(2.6)^x$

4. Explain how the numbers in your function are related to the data.
   To get the first number, I backed up to the term before 3^2 to get 2^2 or 4 for the initial value. The base is the square of the golden ratio since we are using squares of the Fibonacci numbers.

5. Would your function be any different if you started with 2^2 instead of 3^2 as the first area? If so, how and why?
   I would have to change the coefficient to 1 because now 2^2 is the term that is paired with x = 1. The ratio would be still be 2.6 but it would not fit the points as well. The first three terms of the Fibonacci sequence don't approximate the golden ratio as well.

Transparency – The Eye of the Beholder

Throughout history and cultures, humans have been attracted to each other in various ways. One level of attraction has to do with a person's physical appearance – particularly the face. Mathematicians, artists, and physicians have studied certain features of the human face and determined that **ratios** of some measurements of features in the so-called "beautiful people" have a value very close to a specific number.

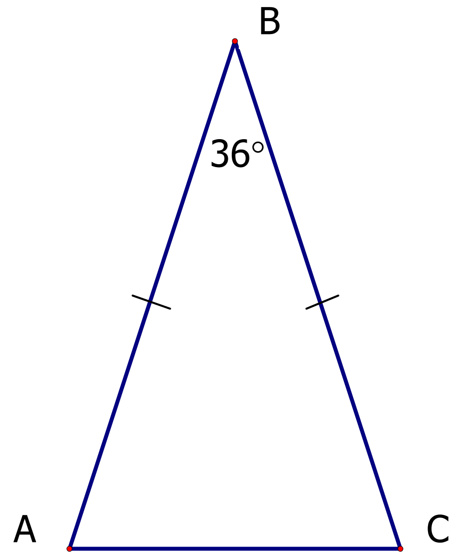


Sketch by artist, Debra L. Hayden, 2005.
Used with permission.

Artists use this so-called "**golden ratio**" to create images that are considered classically beautiful. Using Geometer's Sketchpad and your activity sheet, you will determine how close to "perfect" a celebrity of your choice seems to be.

Transparency 1 – The Golden Triangle

One geometric model of the **golden ratio** is an isosceles triangle with vertex angle of 36°.

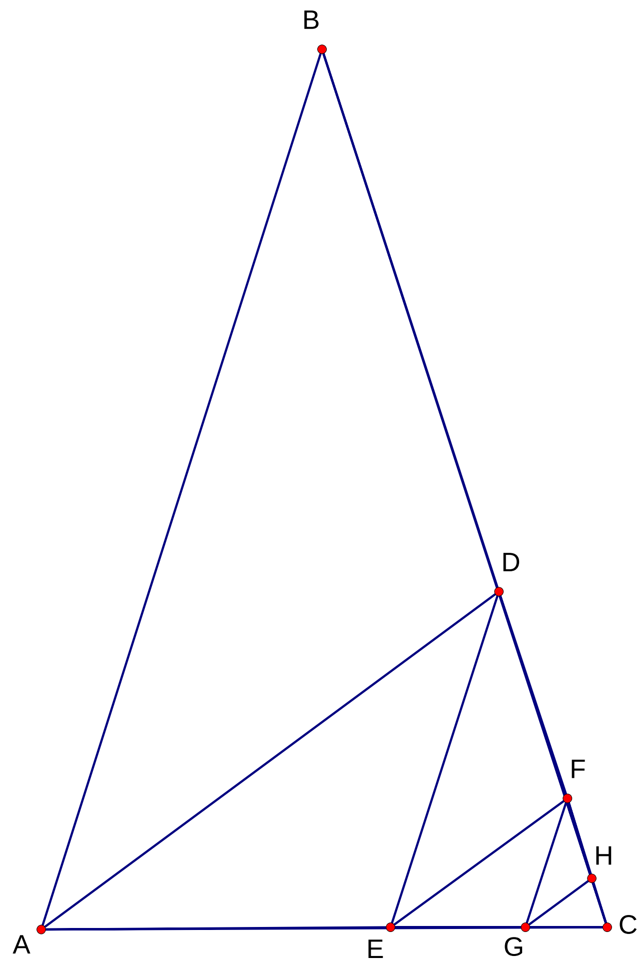


Bisect $\angle A$ and name the point where the bisector intersects $\overline{BC}$ point D. What is true about ΔCAD? How are ΔCAD *and* ΔABC related?

Use Geometer's Sketchpad and the sketch **Golden Triangle** (Golden Triangle 1 tab) to determine the ratios $\frac{\overline{BC}}{\overline{AC}}$ and $\frac{\overline{BD}}{\overline{DC}}$.

Transparency 2 – The Golden Triangle

Repeat the process of bisecting a base angle several times. The results are shown here. See if you can identify the pairs of segments that fit the golden ratio.



Use the sketch **Golden Triangle** (Golden Triangle 2 tab) to find ratios for additional triangles. Record your results on the activity sheet **Creating a "Golden" Exponential Function.**

Transparency 3 – The Golden Section

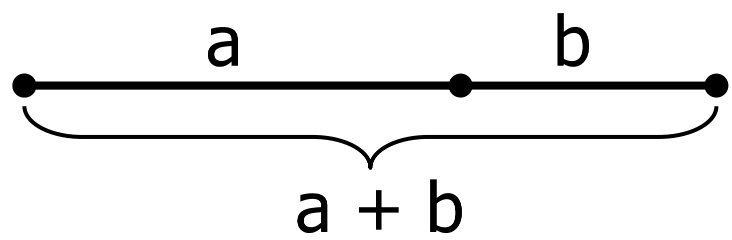
Consider the sequence of numbers:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, …

After the first number, the ratios of any number to the preceding number eventually approximate what we call the "golden ratio". The sequence of numbers was discovered by Leonardo Fibonacci around 1200 A.D.

Transparency 4 – The Golden Section

In geometry, if we take a segment and cut it to represent the golden ratio, it would look like this.



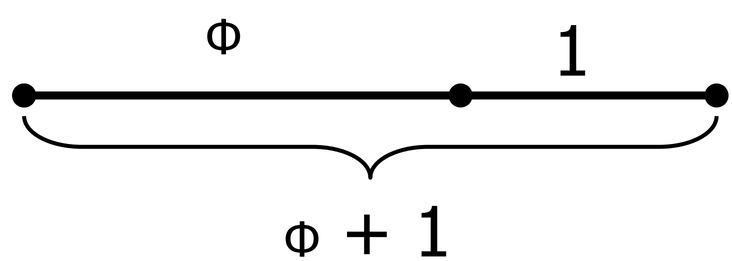
The ratio of the longer section to the shorter section is equal to the ratio of the whole segment to the longer section.

As a proportion, it looks like this:

$$\frac{a}{b} = \frac{a+b}{a}$$

Transparency 5: The *Algebra* of the Golden Ratio

Consider the proportion, $\frac{a}{b} = \frac{a+b}{a}$. The golden ratio **is** the value of $\frac{a}{b}$, but how can we find that value numerically? If we go back to the divided segment and start with the shorter section equaling 1, then the proportion becomes simpler to solve. We will also substitute the symbol, Φ (Greek letter phi), for the larger section.



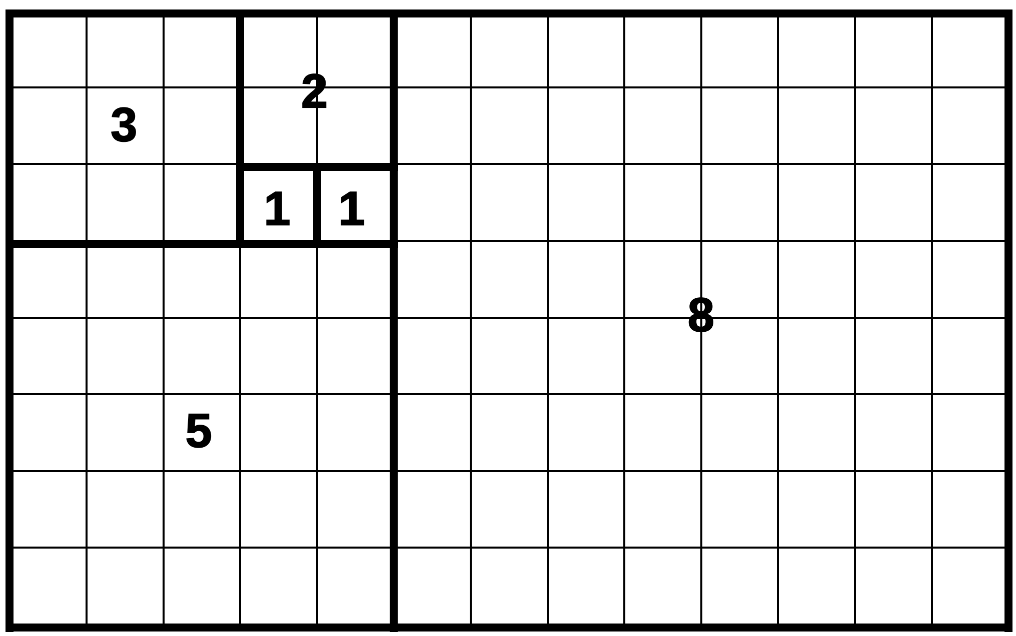
Transparency 6:
The *Algebra* of the Golden Ratio

Now the proportion is $\frac{\Phi}{1} = \frac{\Phi + 1}{\Phi}$. Multiply the means and extremes to get $\Phi^2 = \Phi + 1$. The definition of the value of phi (Φ), the golden ratio, is a number whose value squared equals its value plus one.

Solve the quadratic equation $\Phi^2 = \Phi + 1$.

Transparency – The Golden Rectangle

If consecutive numbers from the Fibonacci sequence were the dimensions of a rectangle, we would have a "golden rectangle." The unique ratios illustrated by a golden rectangle are said to be the most visually aesthetic of all ratios. People often use the numbers of the Fibonacci sequence to create a golden rectangle. Here's one way to look at it. Each square has a Fibonacci number side length. Extend a side to create a length using the next Fibonacci number and you have a golden rectangle.

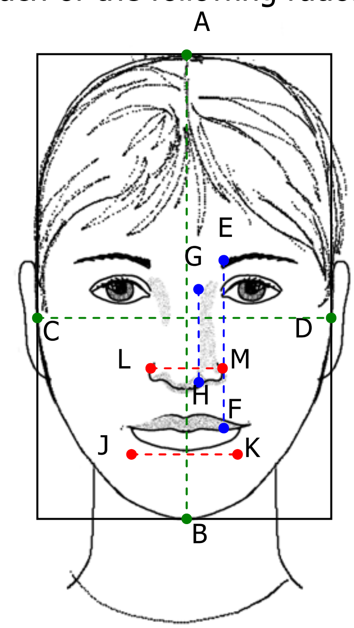


Can you see the rectangles with dimensions of 3×5, 5×8, and 8×13?

Look on the Internet for examples of how artists and architects have used the golden ratio in a rectangular format.

The Eye of the Beholder

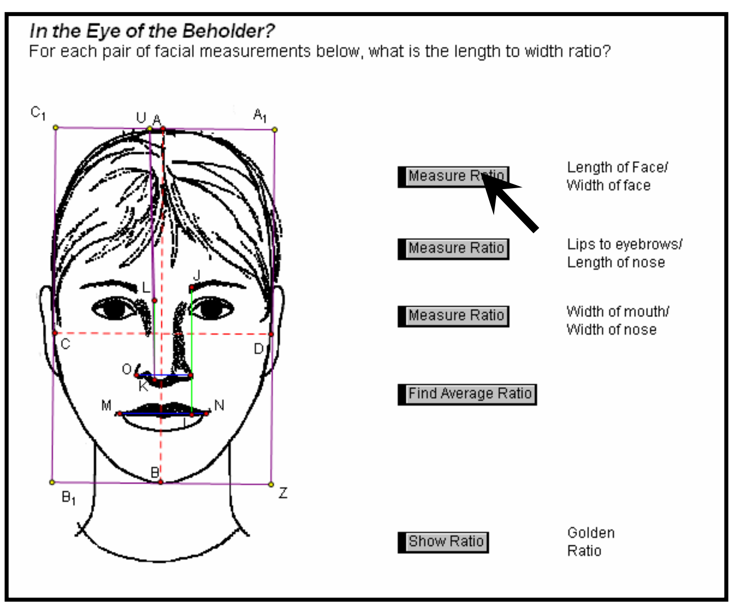
1. Study the features on the artist's sketch below. Identify the segments that represent each of the following ratios.



Sketch by artist, Debra L. Hayden, 2005.
Used by permission

Ratio	Segments
Length of face to Width of face	$\frac{\text{Length of face}}{\text{Width of face}} =$
Lips to eyebrows to Length of nose	$\frac{\text{Lips to eyebrows}}{\text{Length of nose}} =$
Width of mouth to Width of nose	$\frac{\text{Width of mouth}}{\text{Width of nose}} =$
Average Ratio	

2. Open the sketch **Face Sample** in Geometer's Sketchpad. Calculate the ratio values indicated in the sketch by clicking on the "Measure Ratio" action button for the given ratio. Also, calculate the average ratio. Record your answers in your table.



3. Log on to the Internet and open the website http://www.angelfire.com/celeb2/celebrityfaces/. Search for a photo of your favorite celebrity. The photo must be a full front view of the face.

4. Right click on the face and select "Copy" so that you can "insert" the photo into Geometer's Sketchpad.

5. Using Geometer's Sketchpad, construct and measure segments of the face you copied as shown on the sample. Measure the appropriate ratios and record them in the chart below.

I used a photo of :			
Length of face		Ratio	
Width of face			
Lips to eyebrows		Ratio	
Length of nose			
Width of mouth		Ratio	
Width of nose			

6. How do your ratios compare with those found by other groups in the class? Why do you think this is so?

Creating a "Golden" Exponential Function

1. Open the sketch **golden triangle1** to find possible measurements for each of the following:

   The length of $\overline{BC}$ = _______________

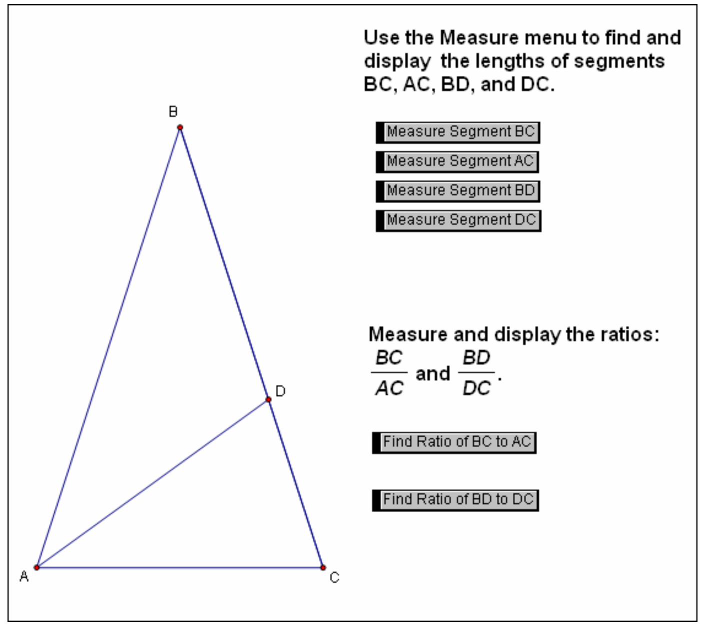
   The length of $\overline{AC}$ = _______________

   The ratio of $\overline{BC} : \overline{AC} \approx$ _______________

   The length of $\overline{BD}$ = _______________

   The length of $\overline{DC}$ = _______________

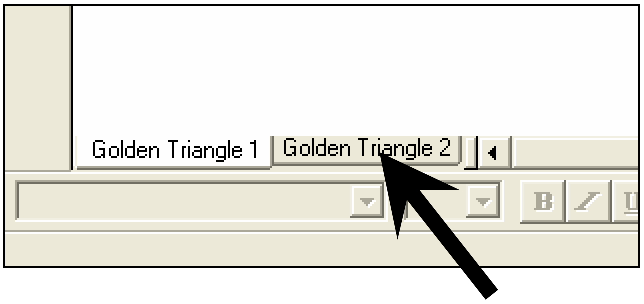
   The ratio of $\overline{BD} : \overline{DC} \approx$ _______________

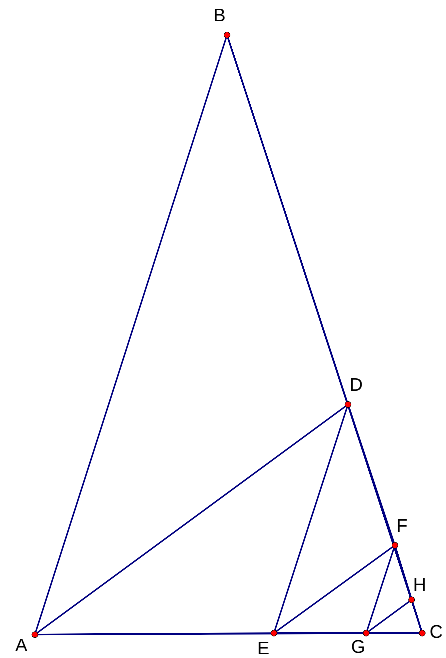


2. Click on Point *C* and drag it around the screen. What happens to the segment lengths?

3. What happens to the ratios when you drag point *C* around the screen?

4. Click the **Golden Triangle 2** tab inside your sketch. Find possible values for the following:





Triangle	Leg	Length	Successive Ratios
1	$\overline{HC}$		
2	$\overline{GC}$		
3	$\overline{FC}$		
4	$\overline{EC}$		
5	$\overline{DC}$		
6	$\overline{AC}$		
7	$\overline{BC}$		

5. Enter the numbers 1 – 7 into List 1 on your graphing calculator. Enter the lengths of the segments $\overline{HC}$, $\overline{GC}$, $\overline{FC}$, $\overline{EC}$, $\overline{DC}$, $\overline{AC}$, and $\overline{BC}$ into List 2. Create a scatter plot on your graphing calculator with the Triangle number on the *x*-axis and the Leg length on the *y*-axis. Sketch your plot and describe your window.

6. Determine an exponential function that passes through these points. Explain how you determined the function.

7. Sketch your plot and function graph. Does the function fit the data well? How do you know?

8. What does the coefficient in your function represent in the golden triangle? How did you obtain this value?

9. What does the base of the power in your function represent in the golden triangle?

Algebra and the Golden Ratio

You have found the exact value of the golden ratio to be $\frac{1+\sqrt{5}}{2}$. Let's look at how this value connects to the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, …

Consider the table below but don't fill in the right-hand column until you've answered questions 1 – 3.

Term number	**Fibonacci number**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

1. If you made a scatter plot of Fibonacci number vs. term number, what would the scatter plot look like?

2. If you started with 1 as your first Fibonacci number, could you write a function that would pass through all of the points in your scatter plot?

3. How could you make a scatter plot that more closely fits an exponential function?

4. Fill in the table with the Fibonacci numbers of your choice and write an exponential function to fit your points.

5. Which would give a better fit: starting with 5 or starting with 13? How does choosing a different starting number affect your function rule?

The Golden Ratio in Art and Architecture

Search the Internet using key words "golden ratio" and "art" or "architecture." Find one example of how the golden ratio is used in art and one example of its use in architecture. Record at least the following information for each example.

Art Example

The artist is/was ______________________________

The name of the painting, sculpture, etc. is ____________________

Give a brief description or simple sketch of how the golden ratio is used in this work.

Architecture Example

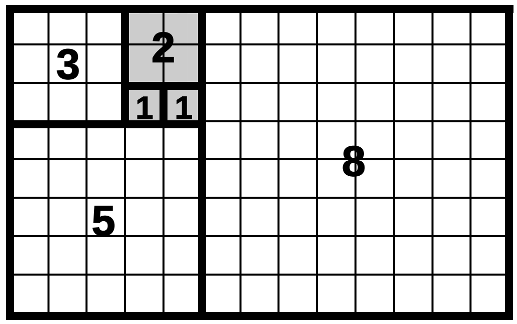
The architect is/was __________________________(or give country where it is located)

The name of the painting, sculpture, etc. is ____________________

Give a brief description or simple sketch of how the golden ratio is used in this structure.

Golden Areas

Consider the squares that make up a golden rectangle shown below. The squares have sides that are terms of the Fibonacci sequence: 1, 1, 2, 3, 5,… Each golden rectangle, such as the square that is shaded, is formed by attaching the next Fibonacci square to the previous golden rectangle.



1. Complete the table below to show the relationship between the number of a square in each golden rectangle and the area of the square. Let the 3 × 3 square be square #1.

Square Number	Area of Square
1	3^2
2	5^2
3	
4	
5	
6	
7	

2. Enter the square numbers into List 1 of a graphing calculator and the areas into List 2. Make a scatter plot for squares 1 – 7. Sketch your scatter plot below and describe the domain and range of the plot.

3. Without using the regression feature on your calculator, write a function that fits your data. Enter your function into your calculator to test it. Alter the function as needed until you are satisfied that it fits the data.

4. Explain how the numbers in your function are related to the data.

5. Would your function be any different if you started with 2^2 instead of 3^2 as the first area? If so, how and why?

1 A stationery company makes cards and posters using dimensions of golden rectangles. So far their inventory includes posters with dimensions (in inches) of 3×5, 5×8, and 8×13. Which equation below would be useful in approximating the length of a poster with a width of 21 inches?

A $L = 13 \times 21$

B $L = 3 \times 1.6^4$

C $L = 13 + 21$

D $L = (1.6)(21)$

2 The table below shows a section of the Fibonacci sequence.

Term number x	Fibonacci number y
0	5
1	8
2	13
3	21
⁝	⁝

Which function best fits the data shown in the table?

A $y = 1.6x$

B $y = 5 * 1.6^x$

C $y = x^{1.6}$

D $y = 8 * 1.6^x$

3 The exact value of *phi*, referred to as the golden ratio, can be found by taking the larger root of the equation $x^2 = x + 1$. What is the exact value of *phi*?

A $\frac{5}{3}$

B 1.618

C $\frac{1+\sqrt{5}}{2}$

D $\frac{1-\sqrt{5}}{2}$

4 The function $y = 2(1.62)^x$ produces the table below when the domain is {1, 2, 3, …}.

X	Y1	
1	3.24	
2	5.2488	
3	8.5031	
4	13.775	
5	22.315	
6	36.151	
7	58.565	

X=1

Which function will produce the table

X	Y1	
1	8.5031	
2	13.775	
3	22.315	
4	36.151	
5	58.565	
6	94.875	
7	153.7	

X=1

for the same domain?

A $y = 1.2346 * 1.62^x$

B $y = 3.24 * 1.62^x$

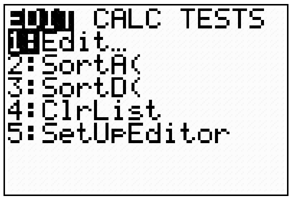
C $y = 5.2488 * 1.62^x$

D $y = 8.5031 * 1.62^x$

Entering and Graphing the Data

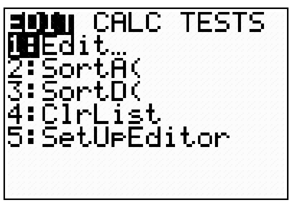
1. Turn the calculator on. Press [STAT] .

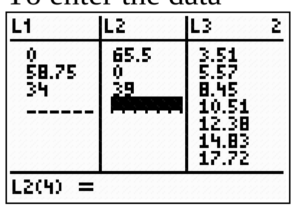
To clear list 1 and list 2, press [2nd] [1] [,] [2nd] [2] [ENTER].



2. Press [STAT] [1]

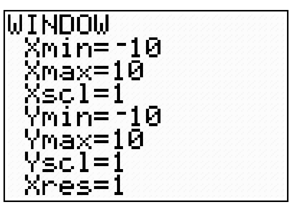
Press [0] [▼] [►] [6] [5] [.] [5] [▼] [◄] [5] [8] [.] [7] [5] [▼] [►] [0] [▼] [◄] [3] [4] [▼] [►] [3] [9] [▼]
To enter the data

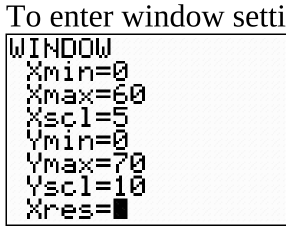




3. Press [WINDOW]

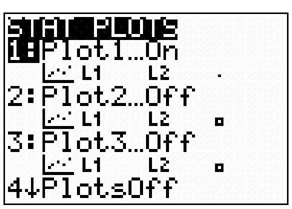
Press [0] [ENTER] [5] [0] [ENTER] [5] [ENTER] [0] [ENTER] [7] [0] [ENTER] [1] [0]
To enter window settings

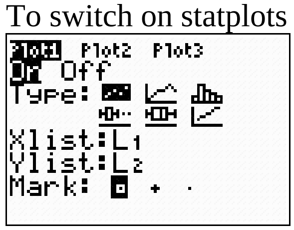




4. Press [2nd] [Y=]

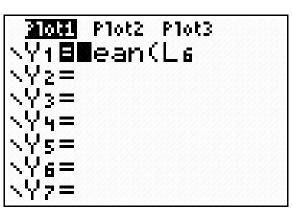
Press [1] [◄] [ENTER] [▼] [ENTER] [▼] [2nd] [1] [▼] [2nd] [2] [▼] [ENTER]
To switch on statplots

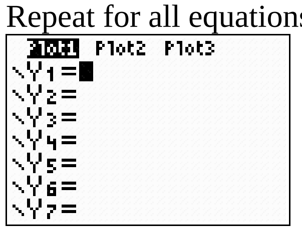




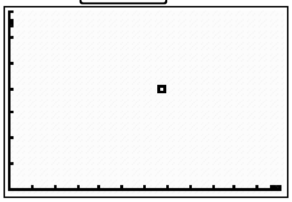
5. Press [Y=]

Press [CLEAR]
To clear equations
Repeat for all equations in Y=



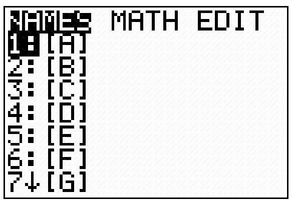


6. Press [GRAPH]

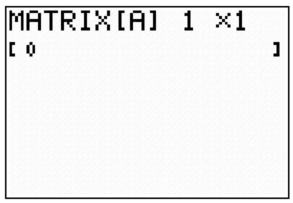


Finding the Model Using Matrices

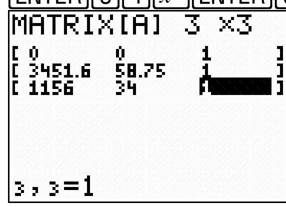
1. Press [2nd][x^{-1}]



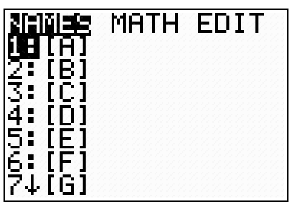
Press [▶][▶][1]
To Edit [A]



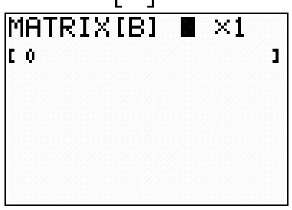
Press [3][ENTER][3][ENTER]
[0][ENTER][0][ENTER][1][ENTER]
[5][8][.][7][5][x^2][ENTER][5][8][.][7][5][ENTER][1]
[ENTER][3][4][x^2][ENTER][3][4][ENTER][1]



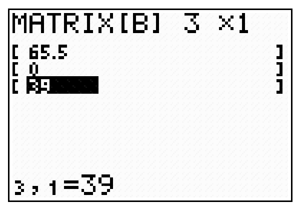
2. Press [2nd][x^{-1}]



Press [▶][▶][2]
To Edit [B]

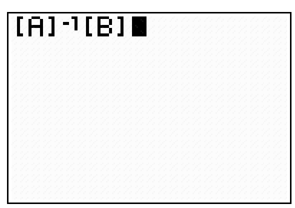


Press [3][ENTER][1][ENTER]
[6][5][.][5][ENTER][0][ENTER][3][9][ENTER]

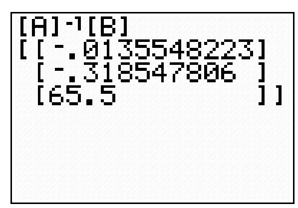


3. Press [MODE]
To go to the Home screen
Press [CLEAR]
To Clear the Home Screen

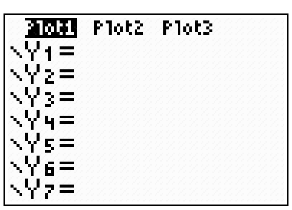
Press [2nd][x^{-1}][1][x^{-1}]
Press [2nd][x^{-1}][2]



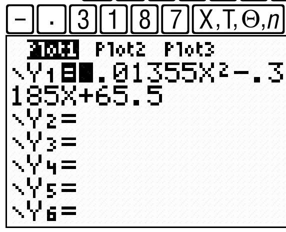
Press [ENTER]



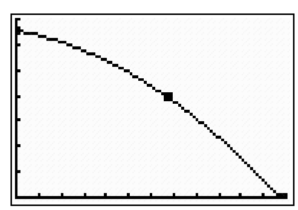
4. Press [Y=]



Press [(-)][.][0][1][3][5][5][X,T,Θ,n][x^2]
[−][.][3][1][8][7][X,T,Θ,n][+][6][5][.][5]

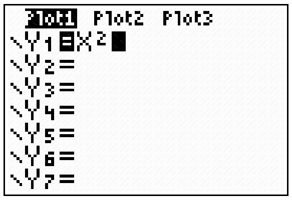


Press [GRAPH]

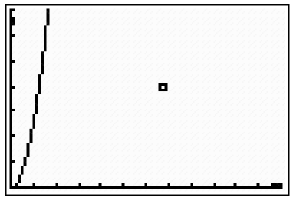


Finding the Model Using Transformations of $y = x^2$

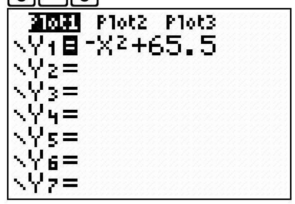
1. Press [Y=] [X,T,Θ,n] [x^2]



Press [GRAPH]



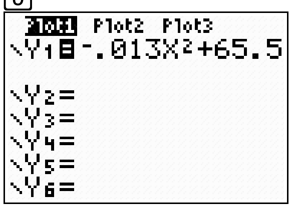
Press [Y=] [(-)] [X,T,Θ,n] [x^2] [+] [6] [5] [.] [5]



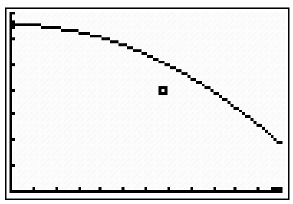
Press [GRAPH]

This process may take many repetitive steps to make the necessary transformations for the model to fit the data. The process has been shortened for this tutorial.

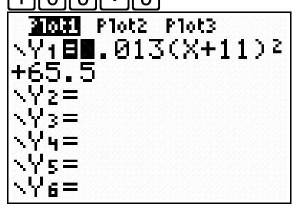
2. Press [Y=] [(-)] [.] [0] [1] [3] [X,T,Θ,n] [x^2] [+] [6] [5] [.] [5]



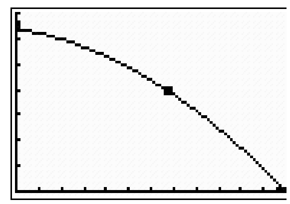
Press [GRAPH]



Press [Y=] [(-)] [.] [0] [1] [3] [(] [X,T,Θ,n] [+] [1] [1] [)] [x^2] [+] [6] [5] [.] [5]



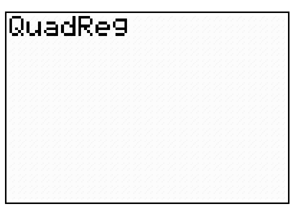
Press [GRAPH]



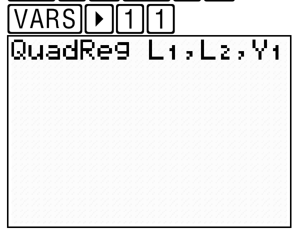
Finding the Model Using Regression

1. Press [STAT]

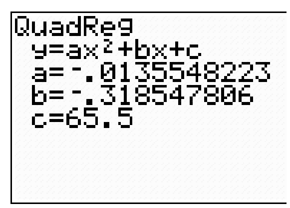
Press [▶] [5]



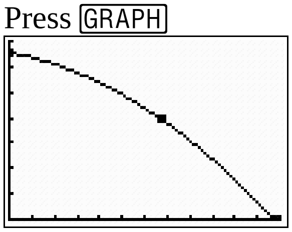
Press [2nd] [1] [,] [2nd] [2] [,] [VARS] [▶] [1] [1]



Press [ENTER]

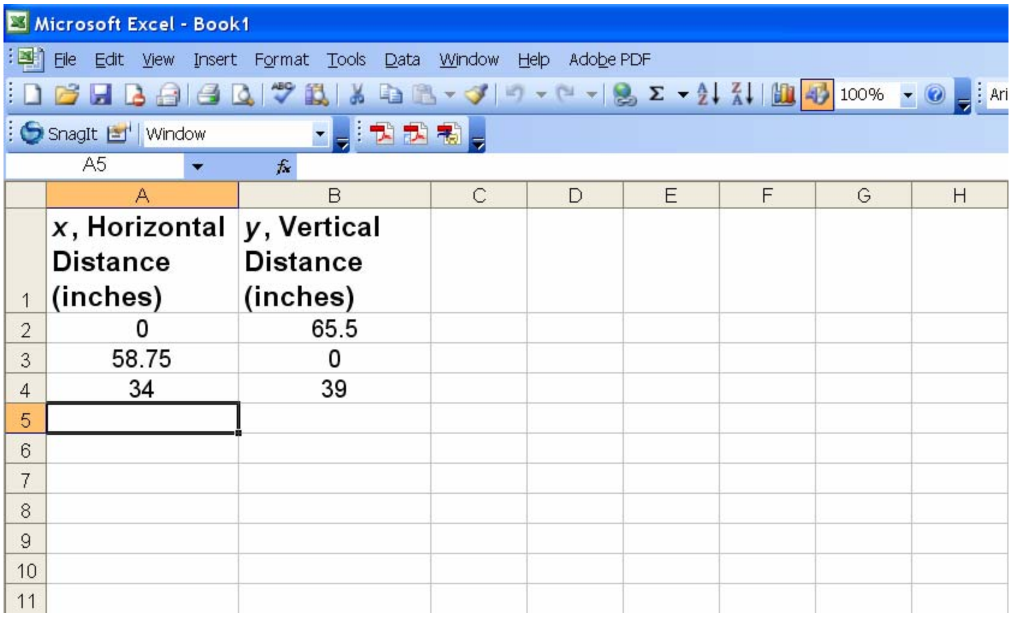


2. Press [GRAPH]

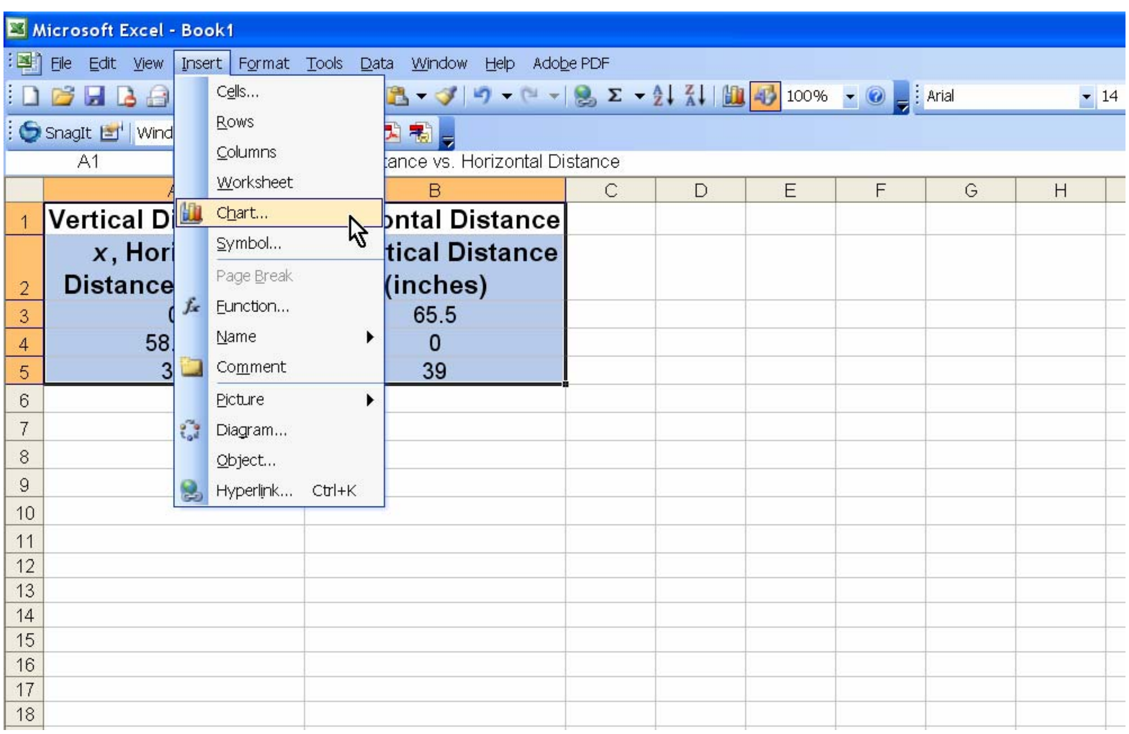


Finding the Model Using Microsoft Excel

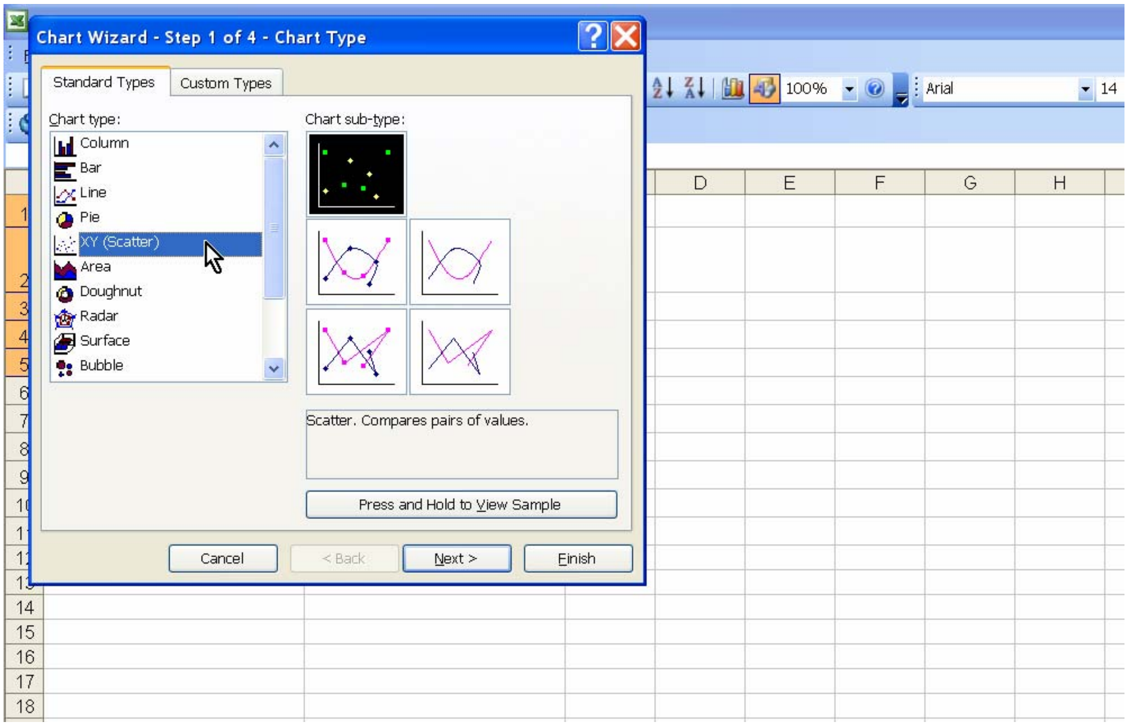
1. Enter column headings and data into the spreadsheet.



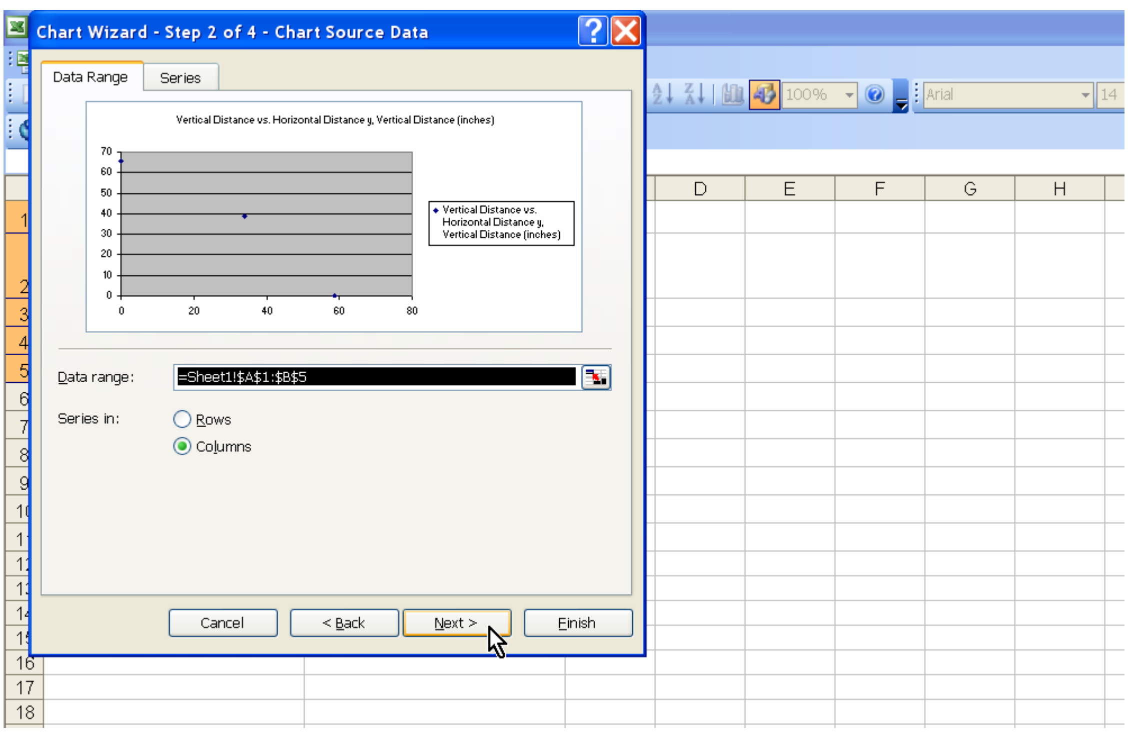
2. Select the data by clicking in the first cell, holding down shift and clicking in the last cell. Next choose **Chart** from the **Insert** menu.



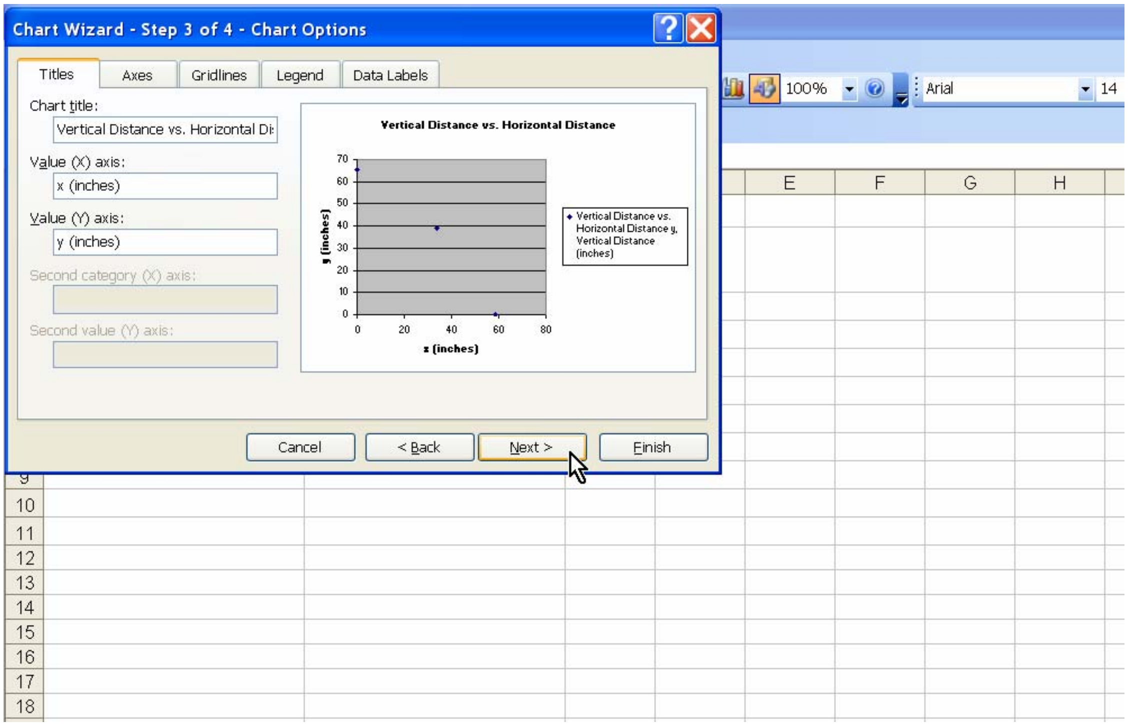
3. Select **XY (Scatter)** then click **Next**.



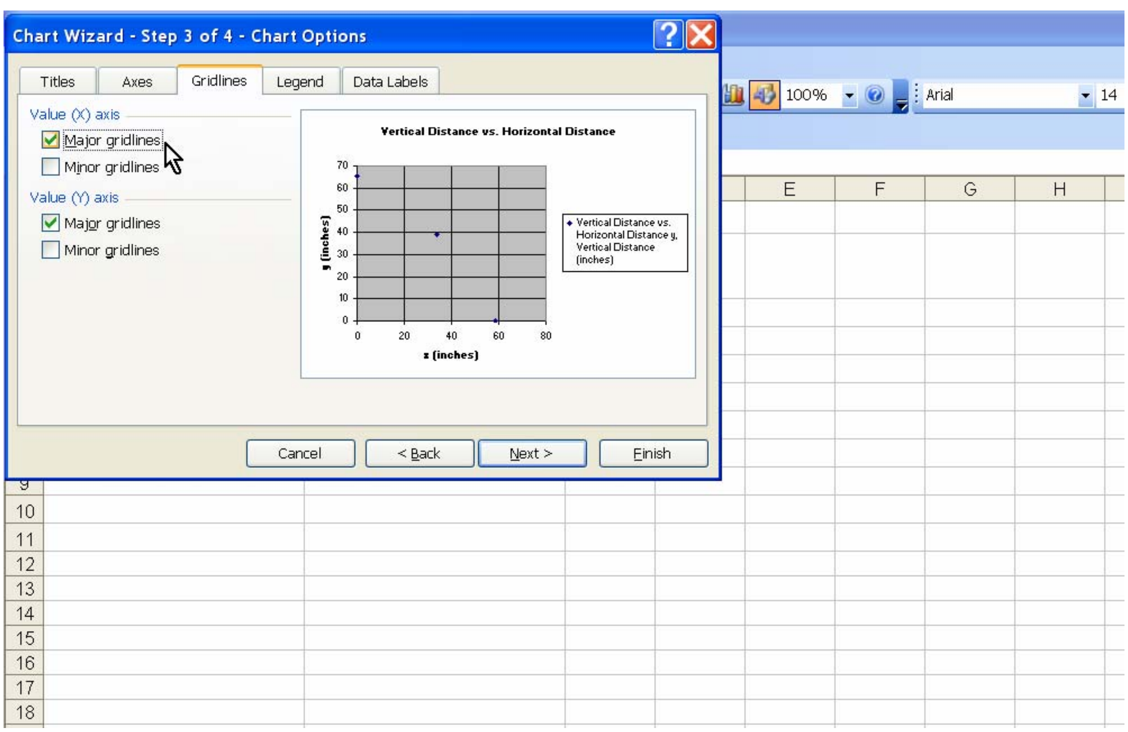
4. Click **Next**.



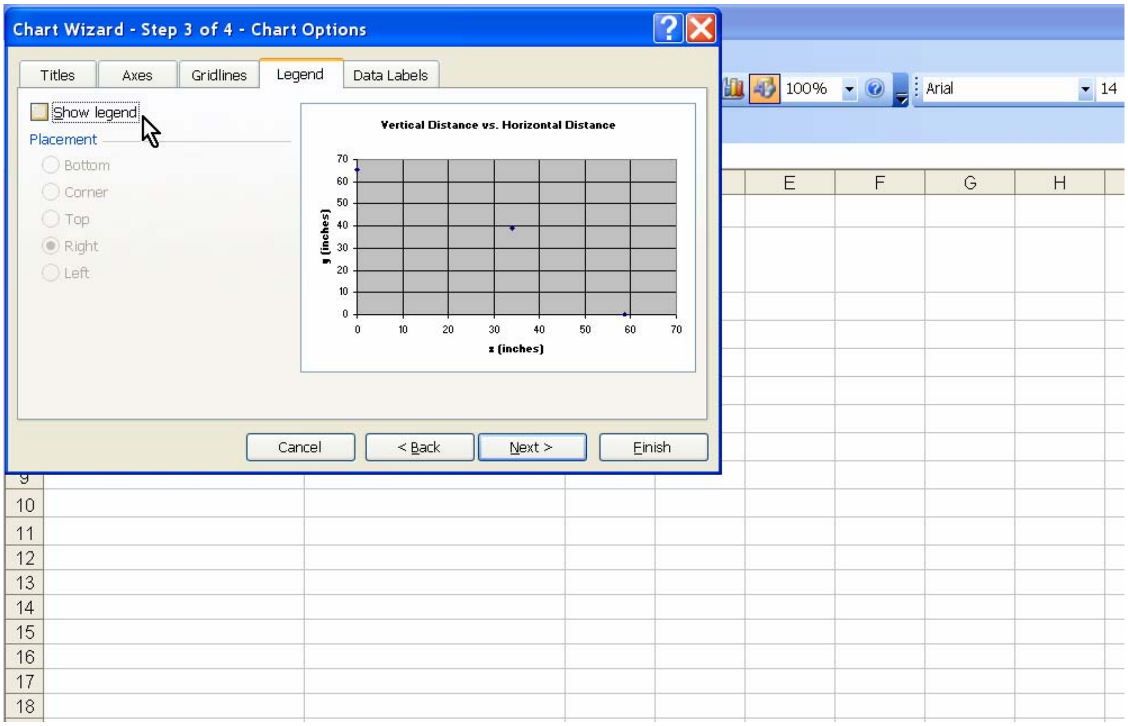
5. Click on the **Titles** tab and enter labels for the *Chart title*, the *Value (X) axis*, and the *Value (Y) axis*.



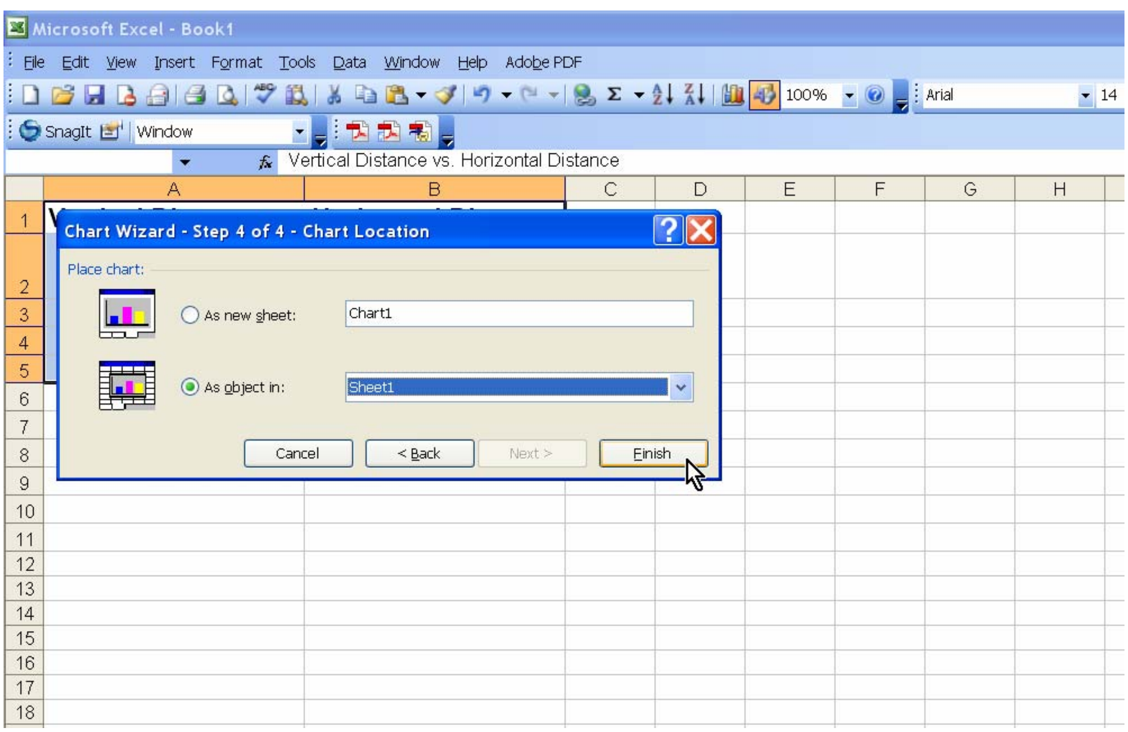
6. Click on the **Gridlines** tab and select the **Major gridlines** under **Value (X) axis**.



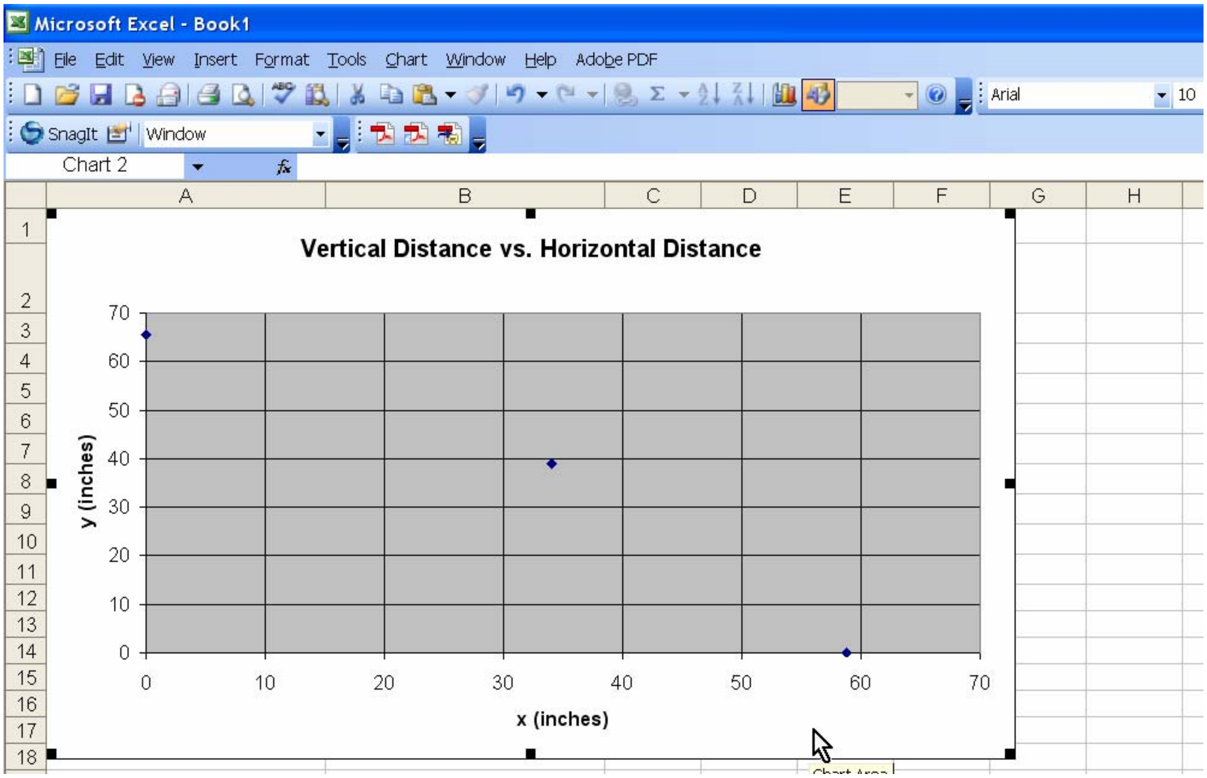
7. Click on the **Legend** tab and deselect **Show legend** then click **Next**.



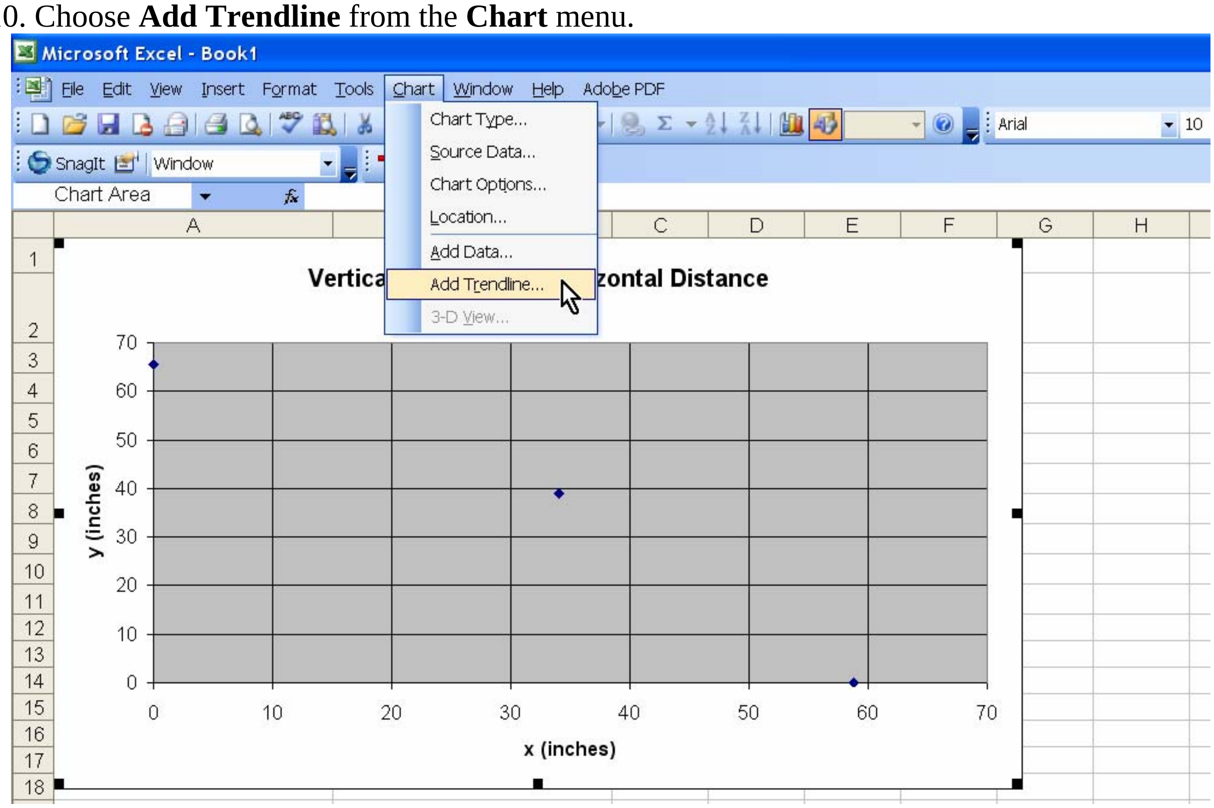
8. Click **Finish**.



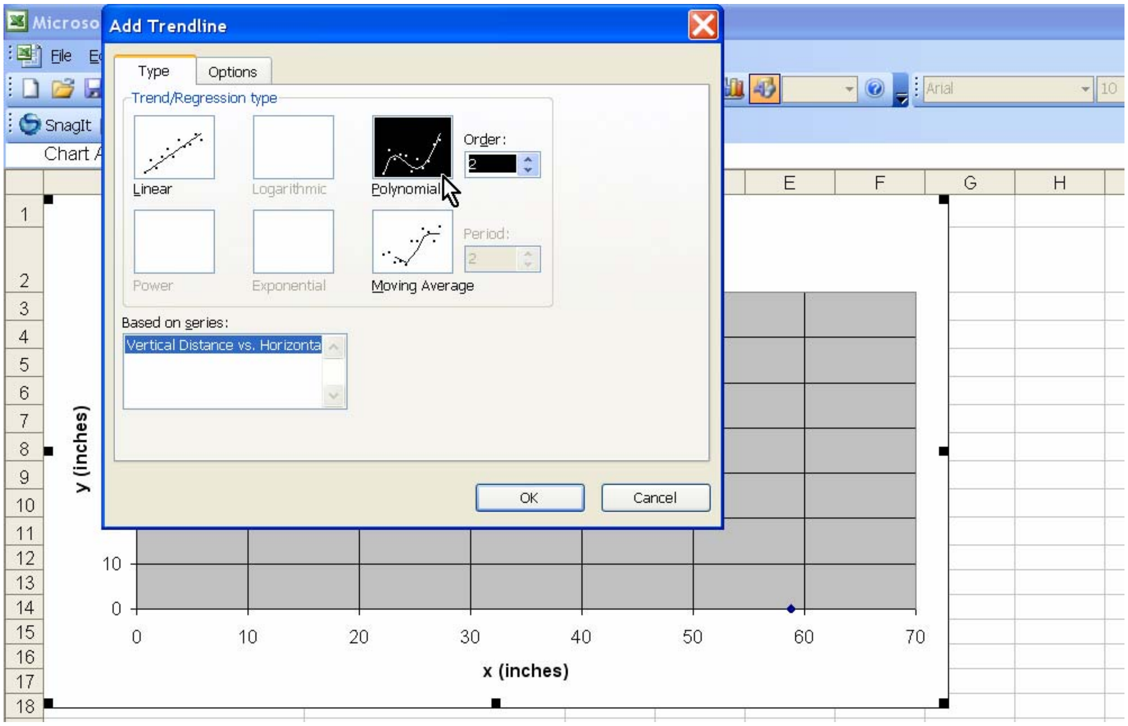
9. Select the chart by clicking on its outer border.



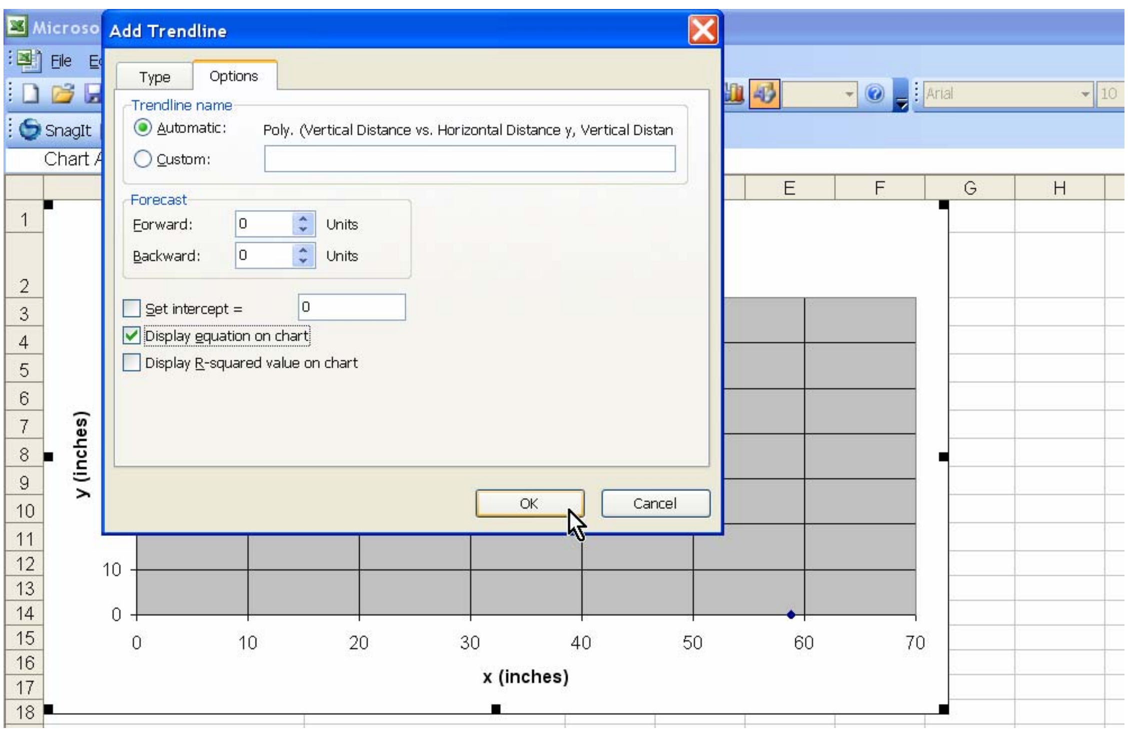
10. Choose **Add Trendline** from the **Chart** menu.

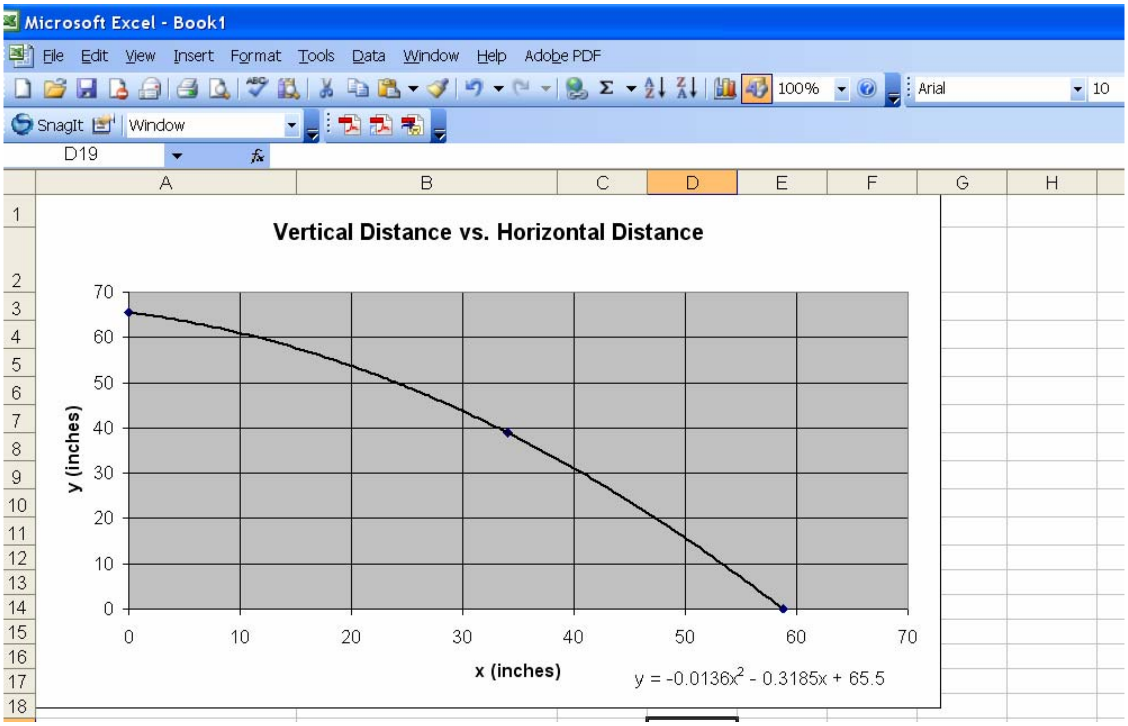


11. Select **Polynomial** and set the **Order** to 2 then click the **Options** tab.



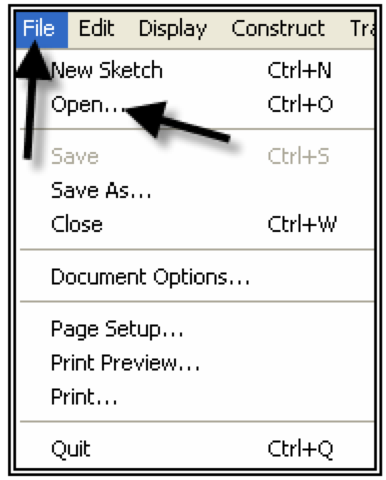
12. Select the **Display equation on chart** check box then click **OK**.



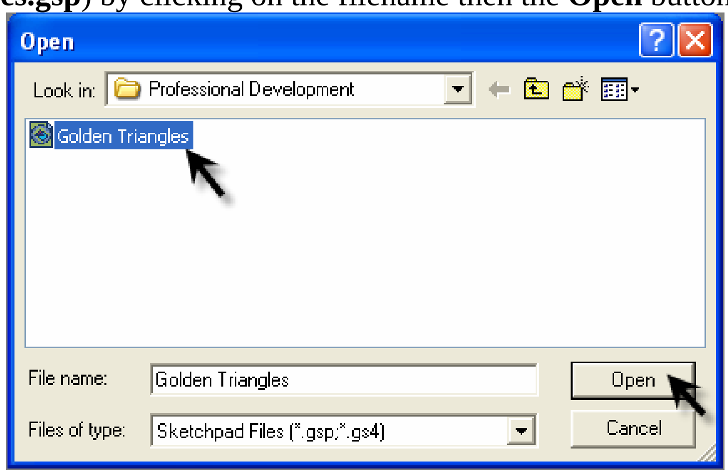

Microsoft Excel - Book1
Vertical Distance vs. Horizontal Distance
y (inches)
x (inches)
y = -0.0136x2 - 0.3185x + 65.5

Opening a Sketch in Geometer's Sketchpad

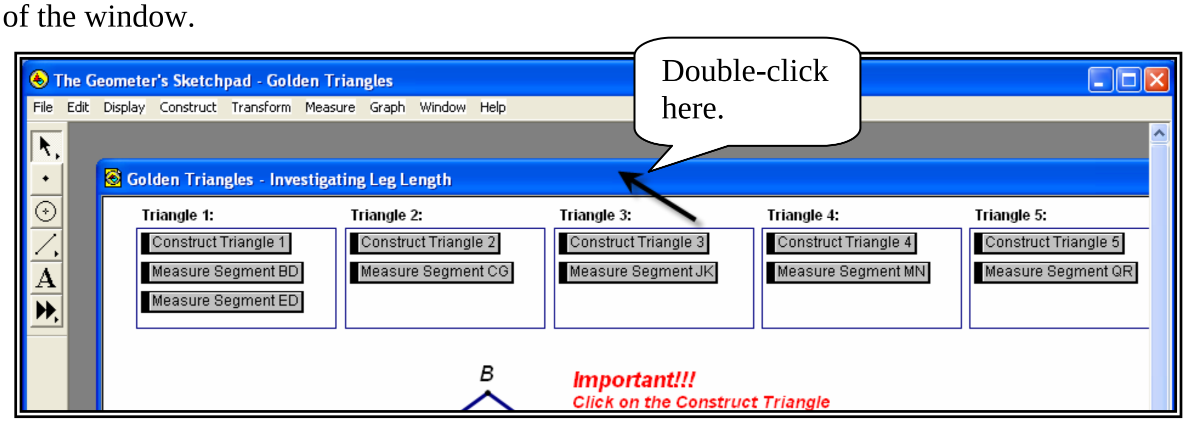
1. To *open* an existing sketch in Geometer's Sketchpad, select **Open** from the **File** menu.



2. A pop up window will appear. Follow the directions for your particular computer system to get to the file where the existing sketches are stored. Select the desired file (in this case, **Golden Triangles.gsp**) by clicking on the filename then the **Open** button.



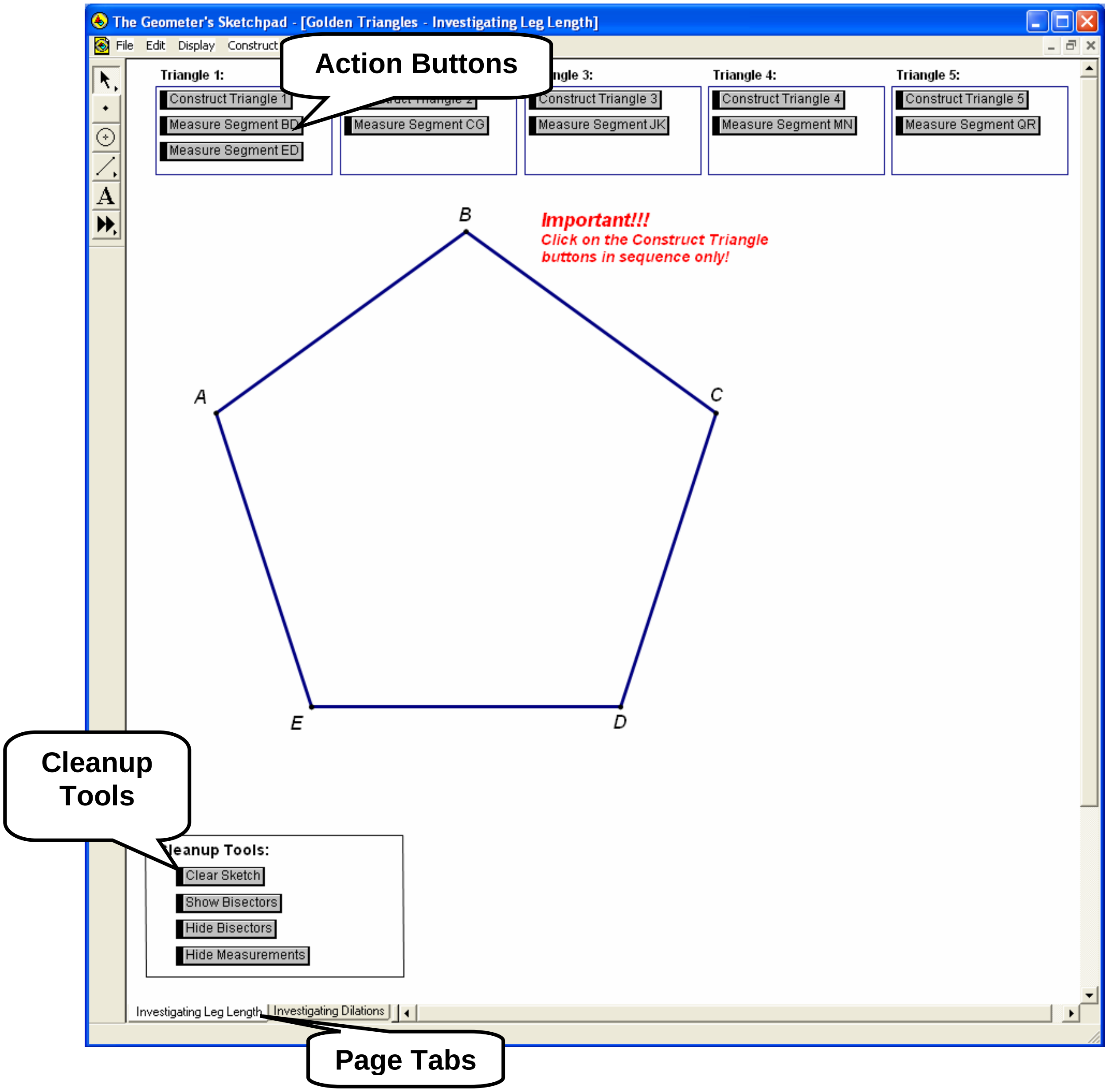
The sketch will open in its own window which you can manipulate like all other windows in Microsoft Windows. To maximize the window, you can double-click on the menu bar at the top of the window.



Working with the "Golden Triangles" sketch:

To work with the "Golden Triangles" sketch, you do not need to be familiar with how to use the Geometer's Sketchpad software. Some features that you may need to know about are:

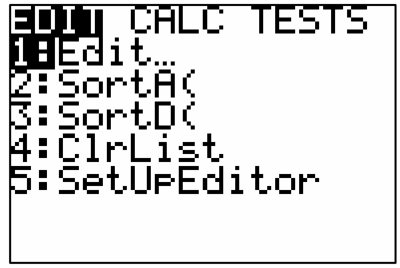
- **Action buttons** are buttons you can click on that cause a particular action to happen. In this sketch, buttons will either construct the next triangle in the sequence or measure a segment length.
- **Cleanup tools** are action buttons that cause certain parts of the sketch to disappear, thus "cleaning up" the sketch.
- **Page tabs** are divider tabs that separate different pages in the sketch. In this sketch, there are two pages: Investigating Leg Length and Investigating Dilations.



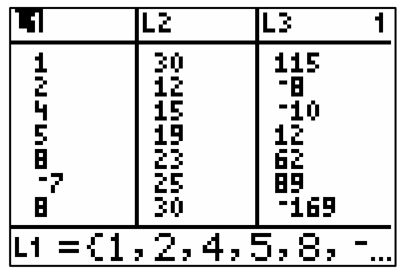
Part 1: Investigating Leg Length

Generating a Scatterplot of Leg Length vs. Triangle Number Using a Graphing Calculator

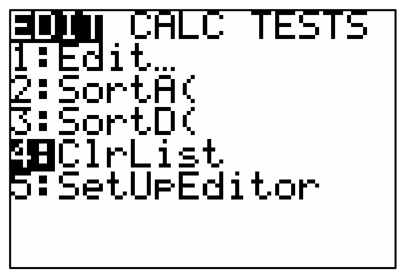
1. Press [STAT]. Then press [ENTER].



2. You will see a table containing lists. Your calculator may contain data in its lists from a previous investigation. If the lists do not contain previous data, you may skip to step 6.



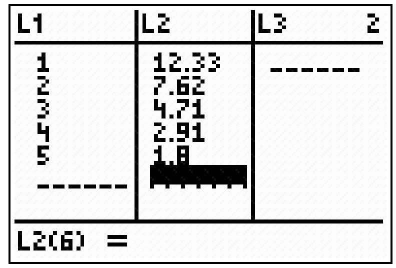
3. To clear this previous data, press [STAT].



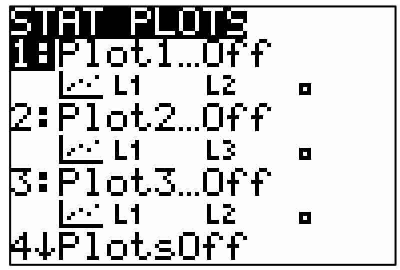
4. Highlight **ClrList**. Enter the lists that you wish to clear. Press [ENTER].

5. Press [ENTER] again.

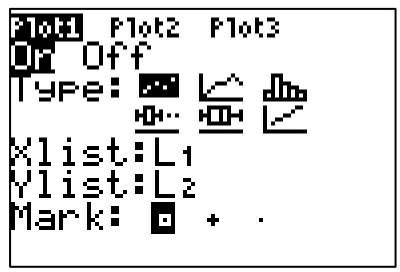
6. Enter the data into the lists.
   Be sure to press [ENTER] after each value.



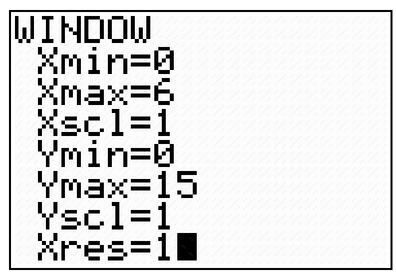
7. Press [2nd] [STAT PLOT].



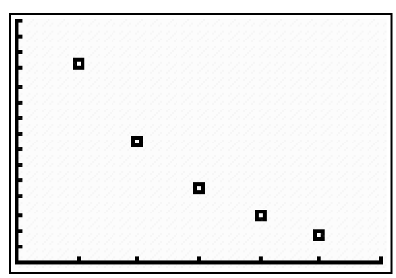
8. Use the arrows to select the necessary options.
   For Plot 1, be sure that the Plot is On and a scatterplot is chosen (first Type). The independent variable (XList) is in L_1 and dependent variable (YList) is in L_2.



11. Choose an appropriate window by selecting [WINDOW] and specifying the appropriate domain and range.
    Use the arrow keys to move up and down.



12. To view the scatterplot, press [GRAPH].



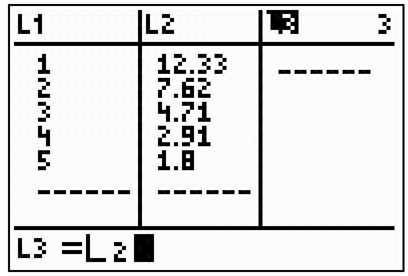
Part 1: Investigating Leg Length

Determining a Function Rule for Leg Length vs. Triangle Number Using a Graphing Calculator

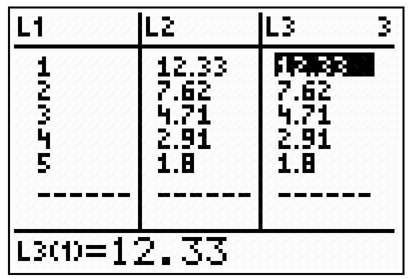
Note: Directions follow for use of a TI-83, TI-83+, or TI-84.

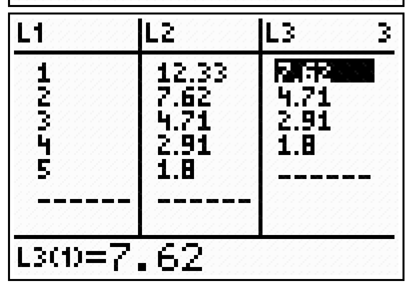
Using Successive Quotients:

1. In the List Editor (Press [STAT] then press [ENTER]), copy List 2 into List 3. To do so, use the arrow keys to move the cursor to the List 3 header, then press [2nd] [2]. Press [ENTER].

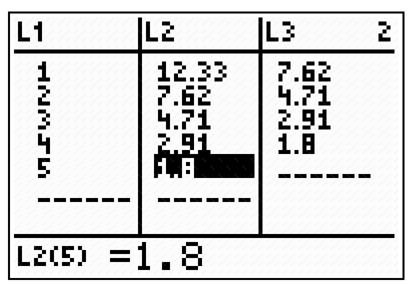


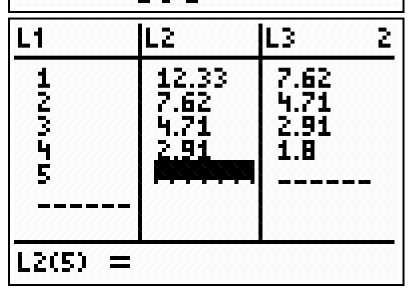
2. Delete the first element of List 3 by using the arrow keys to select it then press [DEL].



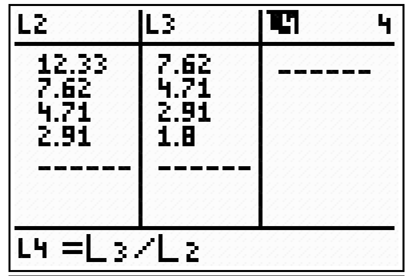


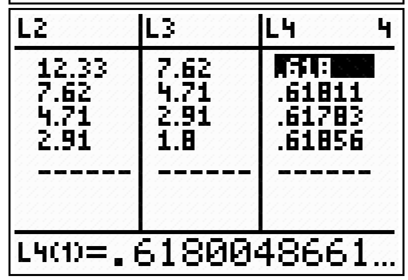
3. Delete the last element of List 2 by using the arrow keys to select it then press [DEL].



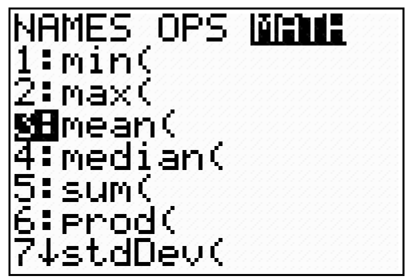


4. Use the arrow keys to select the List 4 header. We want List 4 to be the quotient of List 3 and List 2. Enter the formula $L_4 = L_3/L_2$ by pressing [2nd][3], [÷], then [2nd][2]. List 4 now contains the successive quotients of the leg lengths, or y-values.

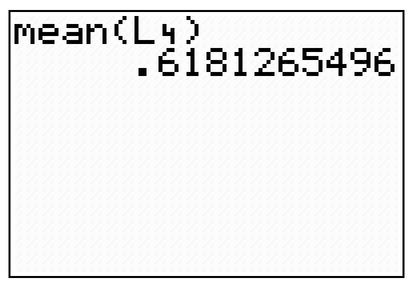




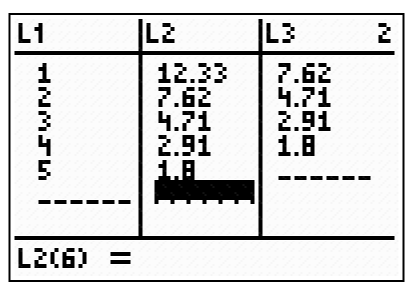
5. Return to the home screen by pressing [2nd][MODE] or [QUIT]. Calculate the mean value of the successive quotients (List 4) by using Math operations on the Lists. Retrieve the List menu by pressing [2nd][STAT] then choose the Math options using the arrow key [▶] twice. Use the down arrow key, [▼], to select option 3: mean.



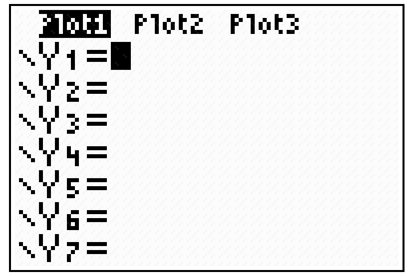
6. Enter the list name for which you want to find the mean value, in this case List 4, by pressing [2nd][4]. Press [ENTER].



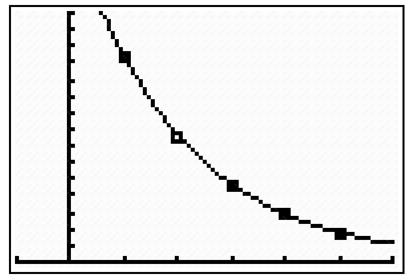
7. Restore the deleted value from List 2. Return to the List Editor (Press [STAT] then press [ENTER]), and use the arrow keys to move to the bottom of List 2. Re-enter the value that you deleted.



8. Use the mean value to determine the values of a and b in the general form $y = a(b)^x$. Graph the function rule that you think might "fit" the data well. To do so, press [Y=]. Clear out any equations by pressing [CLEAR].

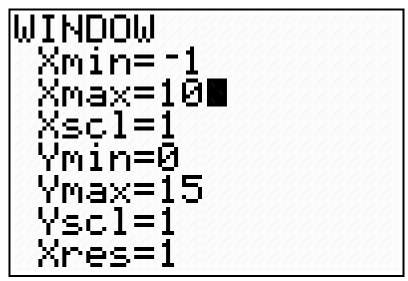


9. Enter the appropriate function rule into Y_1. Press [ENTER]. Press [GRAPH].

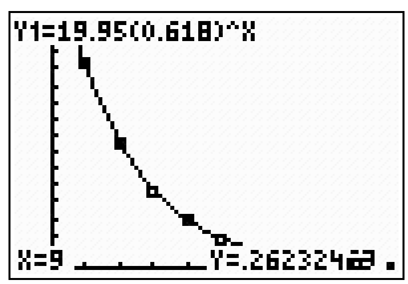


Using the Graph to Make Predictions

1. Press [WINDOW] to enlarge the window. Adjust the settings to make the window large enough to predict with.

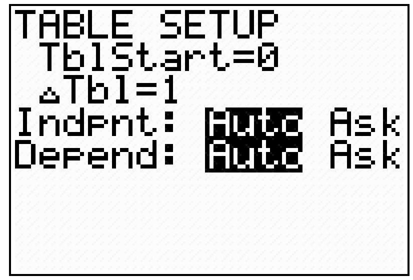


2. Press [GRAPH] then [TRACE]. Press [▲] to select the function then trace to the prediction using the right and left arrow keys, [◄][►].

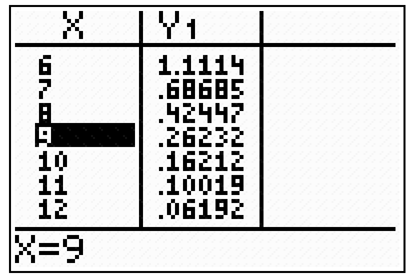


Using the Table to Make Predictions

1. Press [2nd] [WINDOW]. Enter values for TblStart and ΔTbl, the value of the *x* increment.



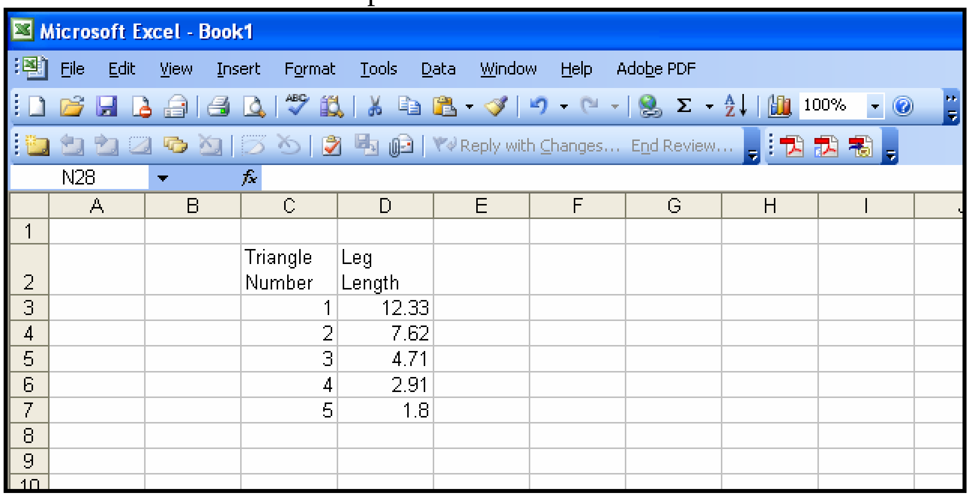
2. Press [2nd] [GRAPH]. Use the up and down arrow keys, [▲] and [▼], to scroll to the desired value.



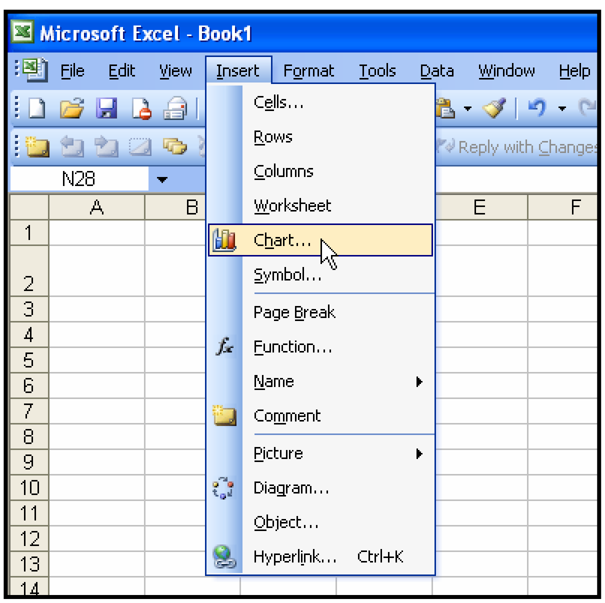
Part 1: Investigating Leg Length

Determining a Function Rule for Leg Length vs. Triangle Number Using a Microsoft Excel Spreadsheet

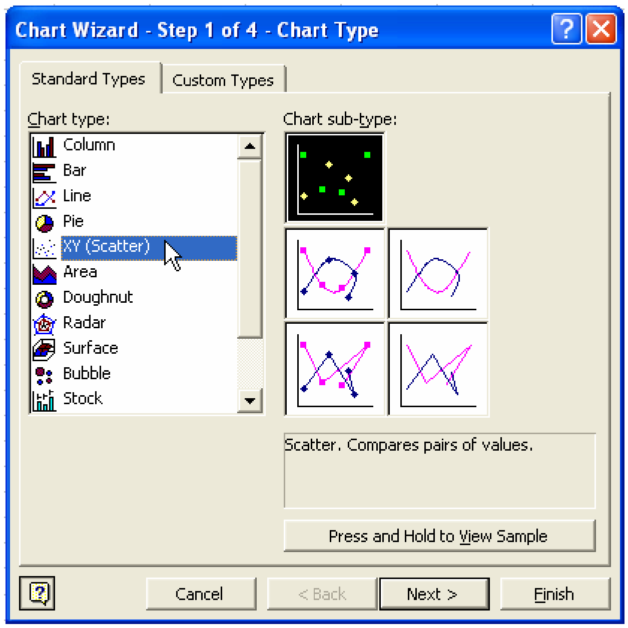
1. Enter your data into a blank Excel spreadsheet.



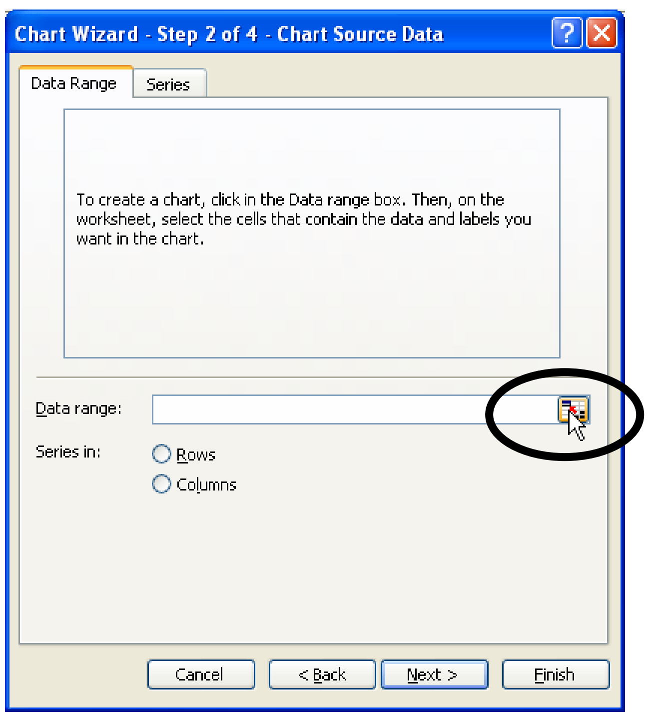
2. Choose **Chart** from the **Insert** menu.



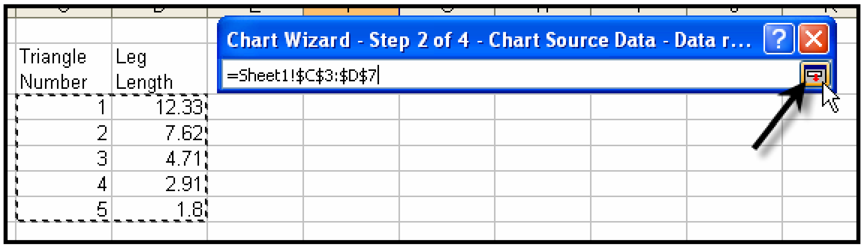
3. Select **XY (Scatter)** from the **Chart Type** selection box then click **Next**.



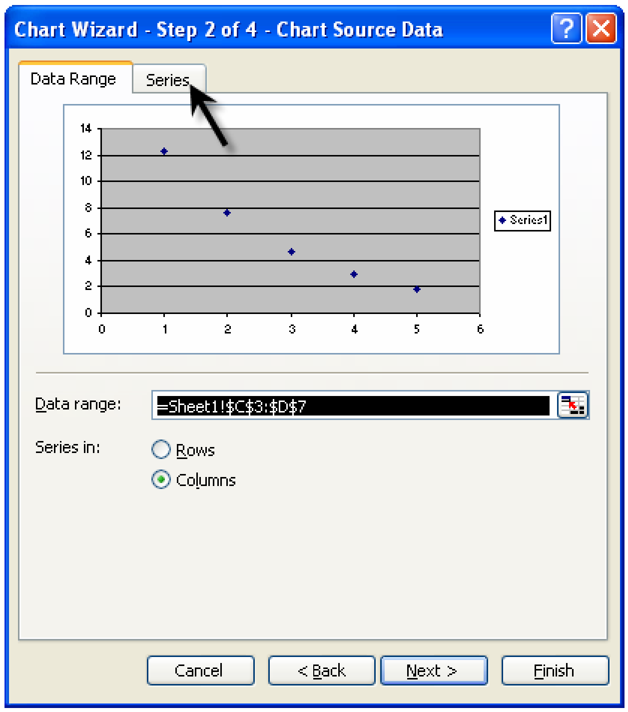
4. To select the Data Range, click the **Collapse Dialog** button next to the **Data Range** text box.



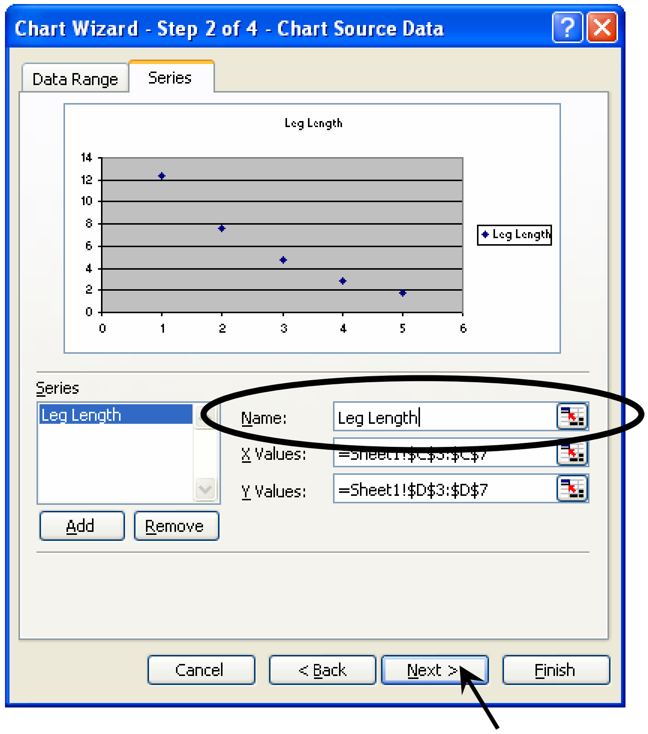
5. Select the cells containing your data then click the **Collapse Dialog** button next to the floating **Chart Source Data** box. You will return to the **Chart Wizard** dialog box.



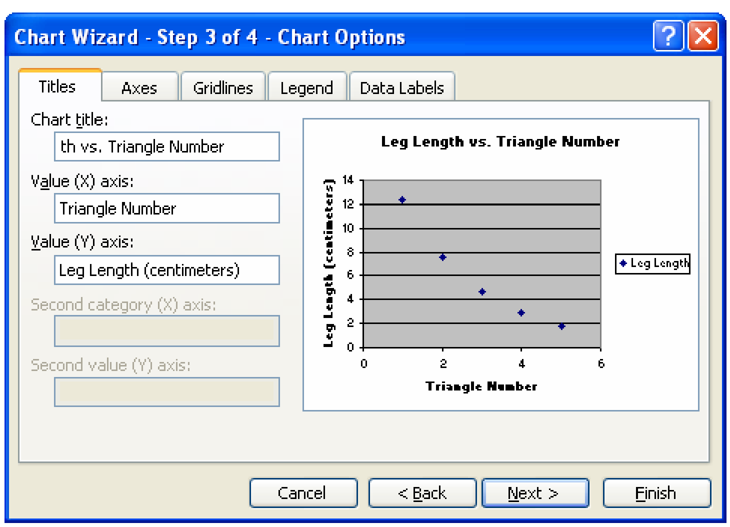
6. Click the **Series** tab in order to edit the source data features.

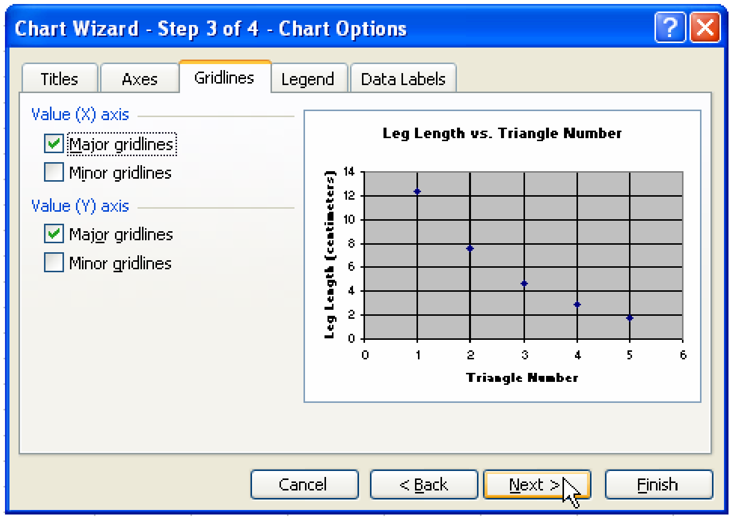


7. Give "Series 1" an appropriate name. Click inside the **Name** text box and type an appropriate name. In this example, we will use "Leg Length." Click **Next**.

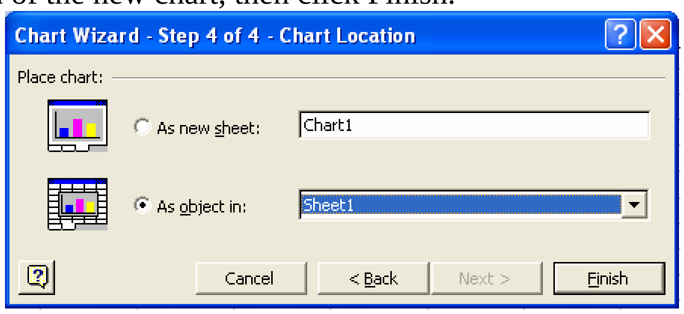


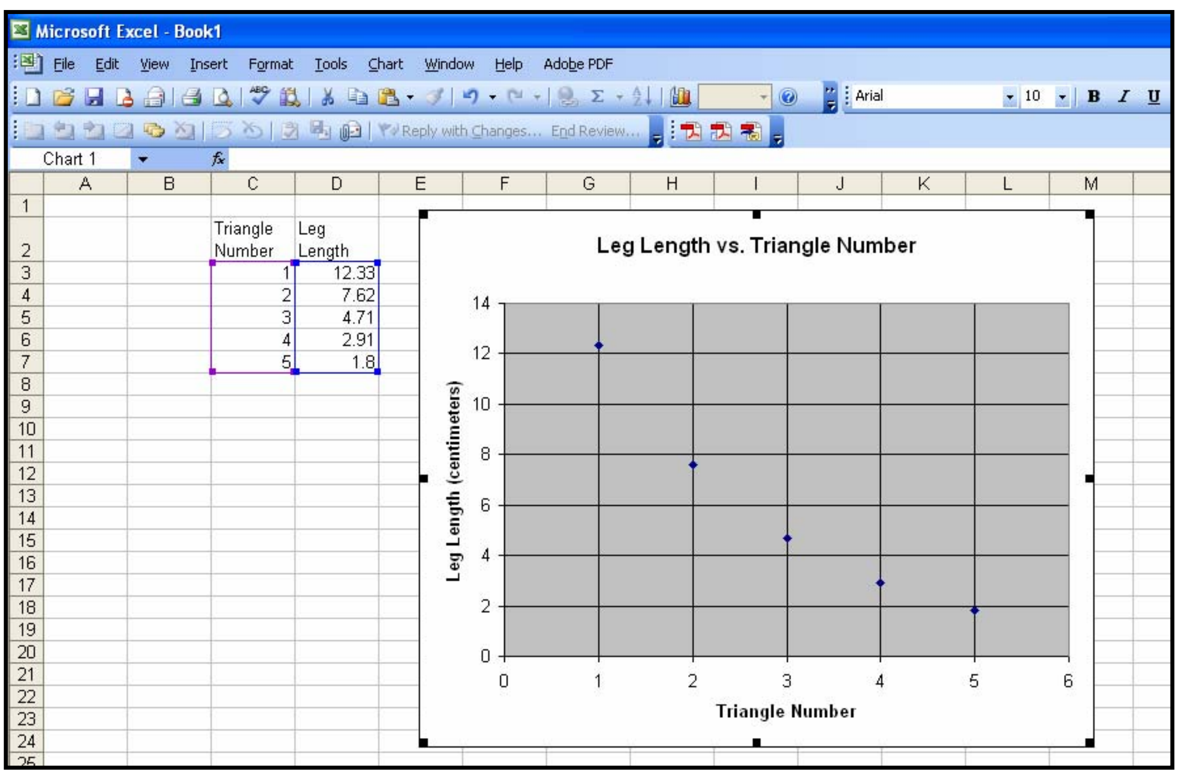
8. At this point you can customize the chart options, including the **Chart title**, **Value (*x*) axis**, and **Value (*y*) axis** labels. Enter the pertinent **Chart Options**, including appropriate labels for the x-axis and y-axis. You can also customize the axes, gridlines, legend, and data labels by clicking on the appropriate tab at the top of the dialog box. Click **Next** when you are ready to continue.





9. Select the location of the new chart, then click Finish.

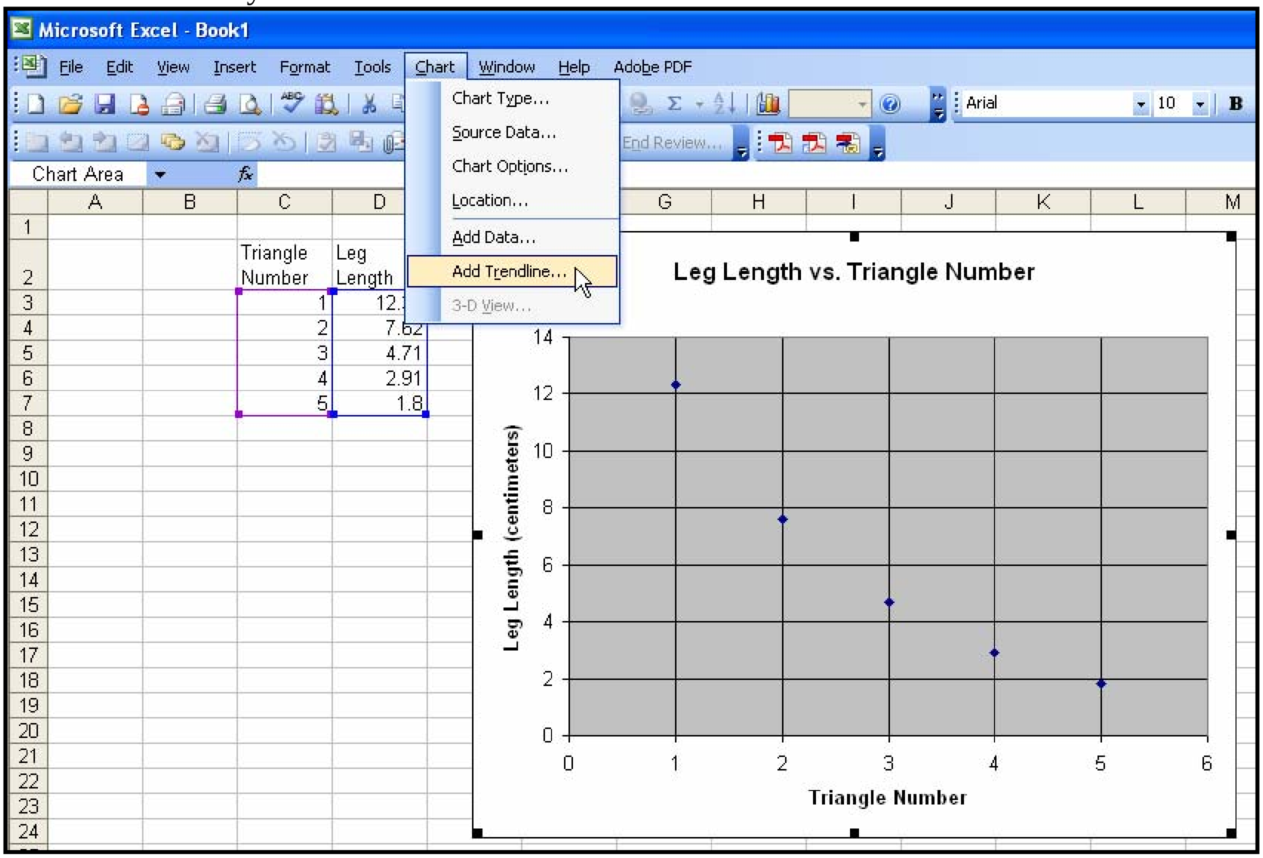




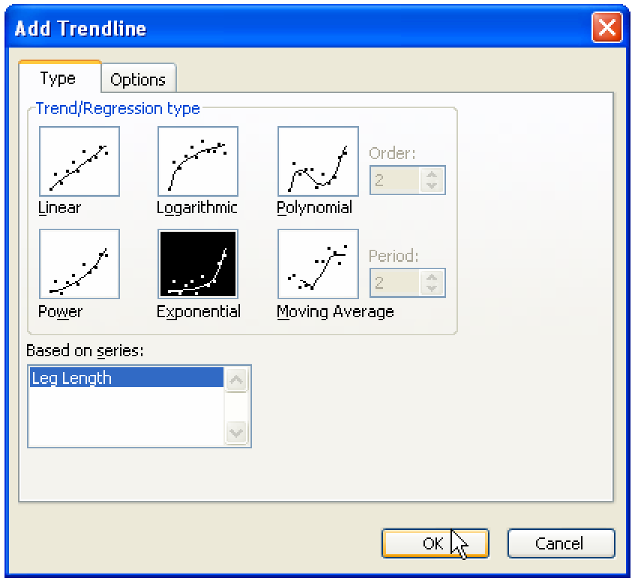
Part 1: Investigating Leg Length

Determining a Function Rule for Leg Length vs. Triangle Number Using a Microsoft Excel Spreadsheet

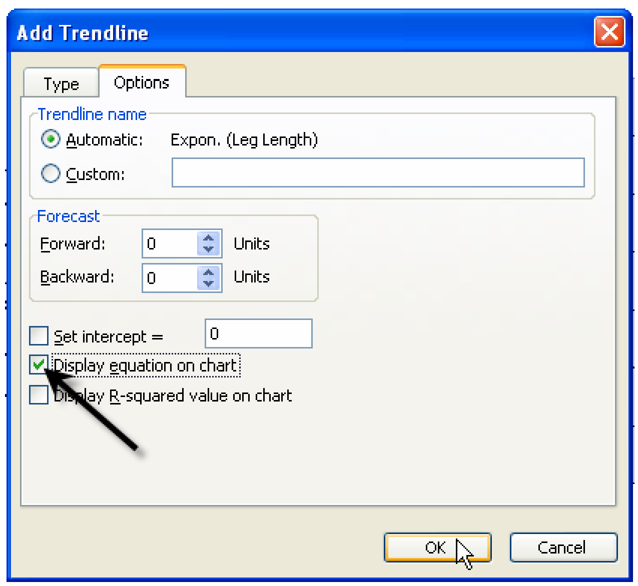
1. Click to select your chart. Choose **Add Trendline** from the **Chart** menu.



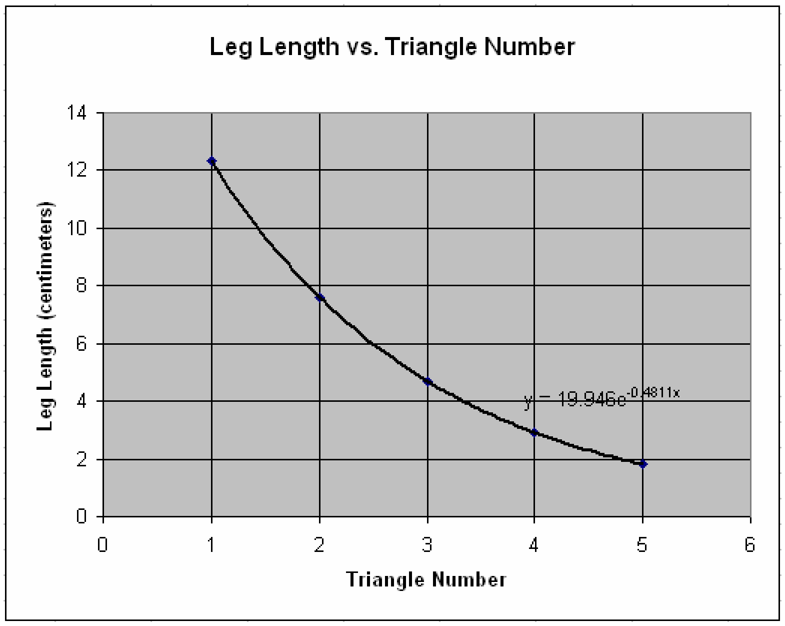
2. The **Add Trendline** dialog box will appear. Click on the **parent function** for the trendline you wish to graph. If you select **Polynomial** or **Moving Average**, be sure to select the order or period, respectively.

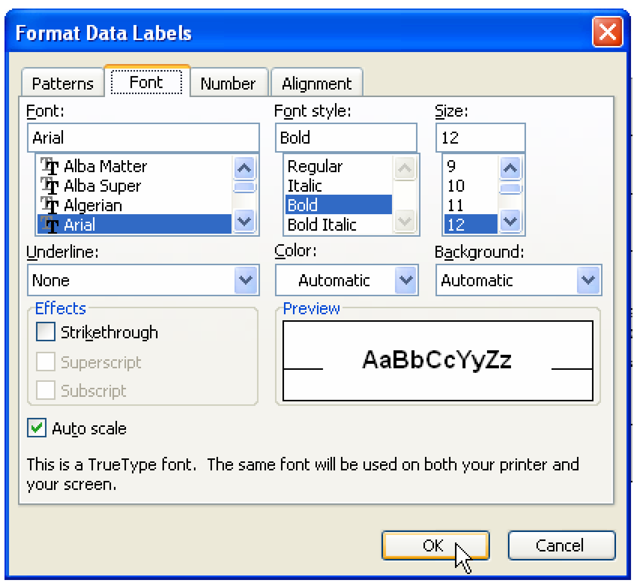


3. Click on the **Options** tab. Click on the **Display equation on chart** check box. Set any other features that you would like to customize related to your trend line. Click **OK**.



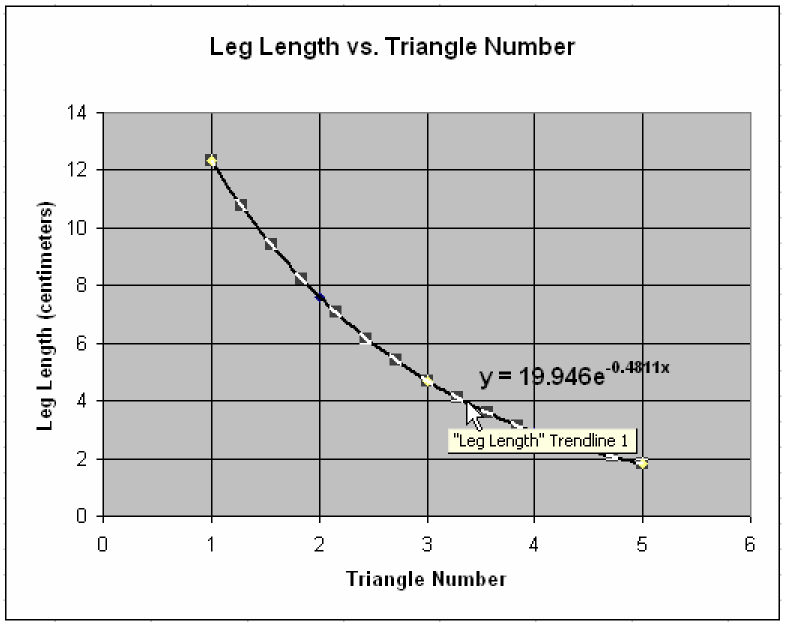
4. Customize the appearance of the equation by double-clicking on the equation. The **Format Data Labels** dialog box will appear. You can change the appearance of the equation, including font, number, and alignment. Click **OK** when you are finished.



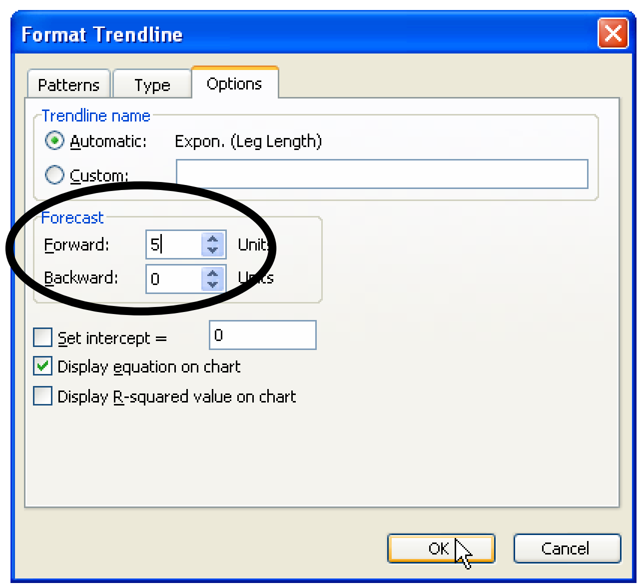


Using the Graph to Make Predictions

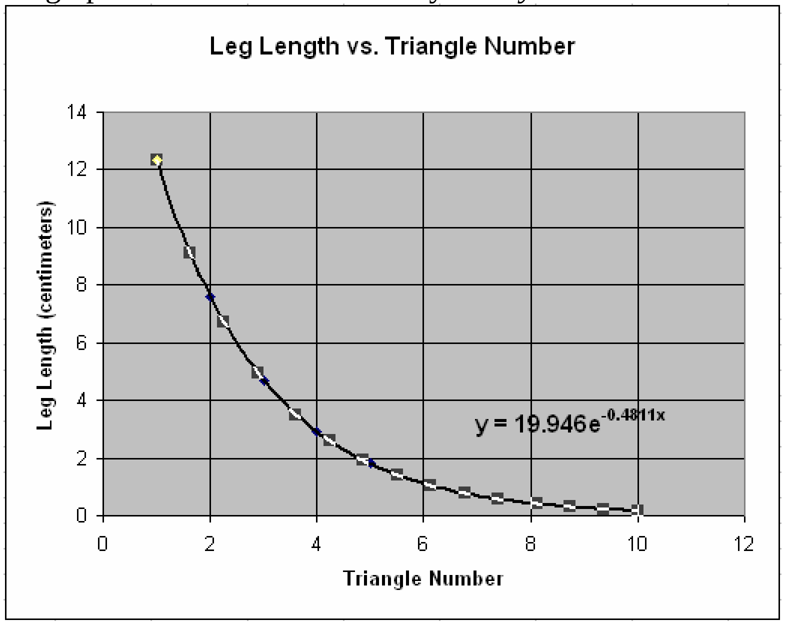
1. Double-click the trendline on your chart. The Format Trendline dialog box will appear.



2. Click the **Options** tab. In the **Forecast** text boxes, enter the number of units that you would like to extend the graph either **Forward** or **Backward** beyond your data set. Click **OK**.



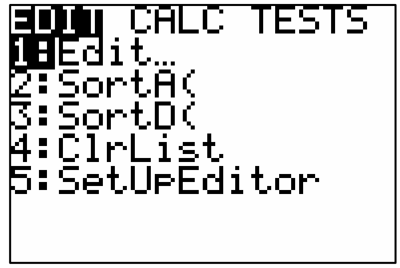
3. Use the extended graph to estimate the necessary *x*- or *y*-value.



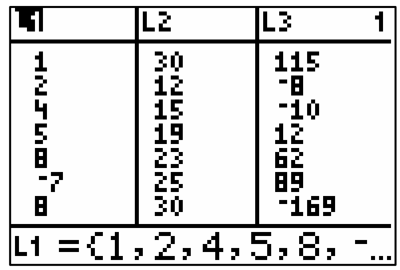
Part 2: Investigating Dilations

Generating a Scatterplot of Leg Length vs. Dilation Number Using a Graphing Calculator

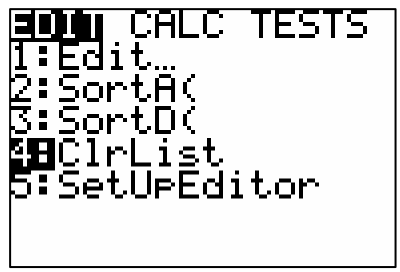
1. Press [STAT]. Then press [ENTER].



2. You will see a table containing lists. Your calculator may contain data in its lists from a previous investigation. If the lists do not contain previous data, you may skip to step 6.



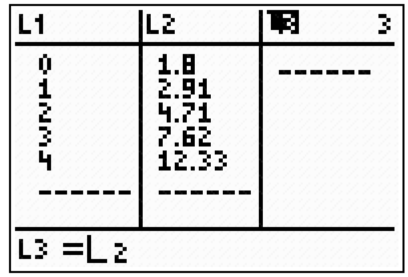
3. To clear this previous data, press [STAT].



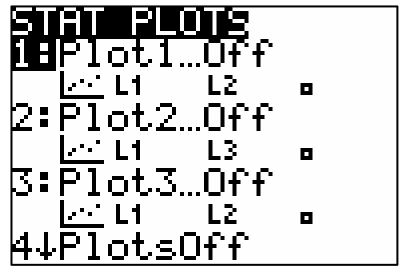
4. Highlight **ClrList**. Enter the lists that you wish to clear. Press [ENTER].

5. Press [ENTER] again.

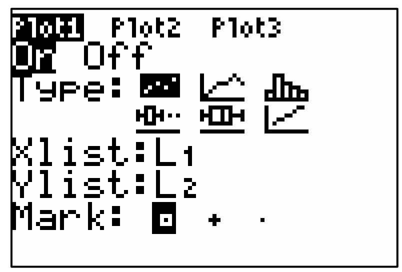
6. Enter the data into the lists.
   Be sure to press [ENTER] after each value.



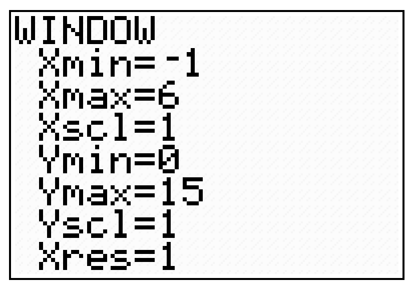
7. Press [2nd] [STAT PLOT].



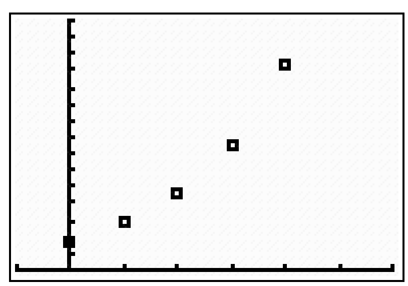
8. Use the arrows to select the necessary options.
   For Plot 1, be sure that the Plot is On and a scatterplot is chosen (first Type). The independent variable (XList) is in L_1 and dependent variable (YList) is in L_2.



11. Choose an appropriate window by selecting [WINDOW] and specifying the appropriate domain and range.
    Use the arrow keys to move up and down.



12. To view the scatterplot, press [GRAPH].



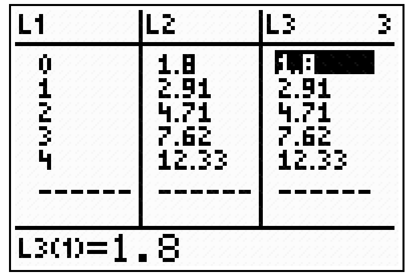
Part 2: Investigating Dilations

Determining a Function Rule for Leg Length vs. Dilation Number Using a Graphing Calculator

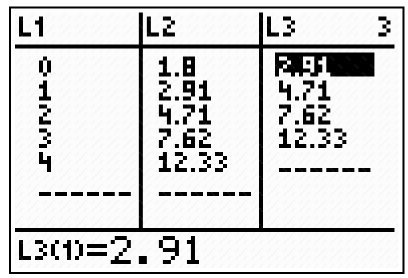
Note: Directions follow for use of a TI-83, TI-83+, or TI-84.

Using Successive Quotients:

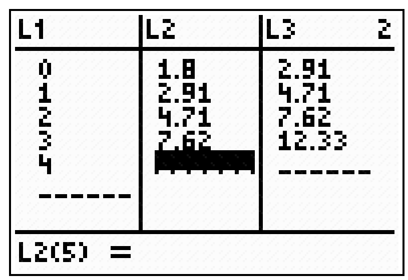
1. In the List Editor (Press [STAT] then press [ENTER]), copy List 2 into List 3. To do so, use the arrow keys to move the cursor to the List 3 header, then press [2nd] [2]. Press [ENTER].



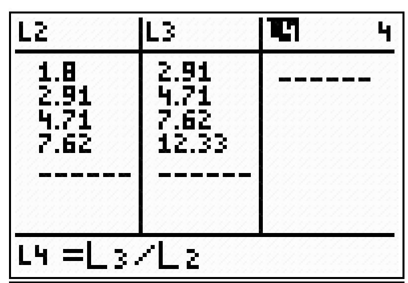
2. Delete the first element of List 3 by using the arrow keys to select it then press [DEL].

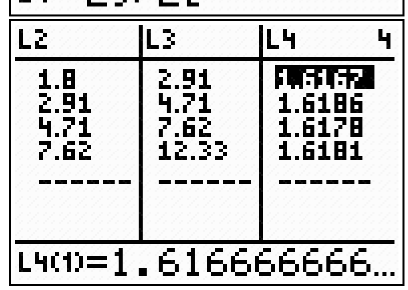


3. Delete the last element of List 2 by using the arrow keys to select it then press [DEL].

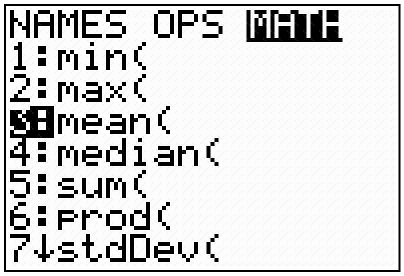


4. Use the arrow keys to select the List 4 header. We want List 4 to be the quotient of List 3 and List 2. Enter the formula $L_4 = L_3/L_2$ by pressing [2nd][3], [÷], then [2nd][2]. List 4 now contains the successive quotients of the leg lengths, or y-values.

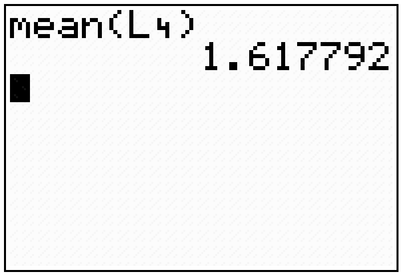




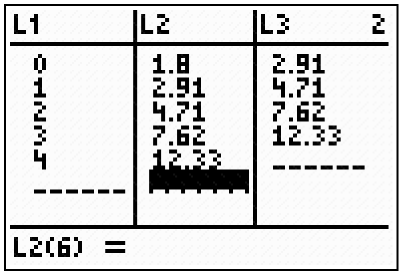
5. Return to the home screen by pressing [2nd][MODE] or [QUIT]. Calculate the mean value of the successive quotients (List 4) by using Math operations on the Lists. Retrieve the List menu by pressing [2nd][STAT], then choose the Math options using the arrow key [▶] twice. Use the down arrow key, [▼], to select option 3: mean.



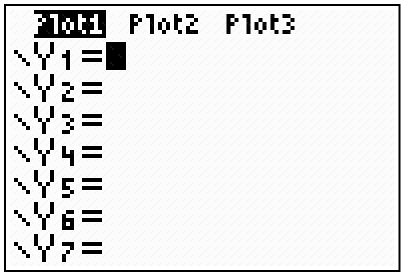
6. Enter the list name of which you want to find the mean value, in this case List 4 by pressing [2nd][4]. Press [ENTER].



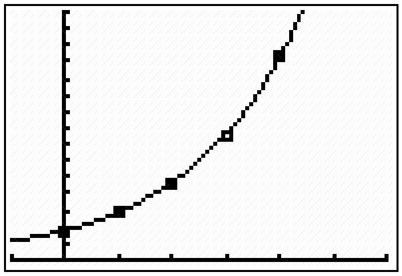
7. Restore the deleted value from List 2. Return to the List Editor (Press [STAT] then press [ENTER]) and use the arrow keys to move to the bottom of List 2. Re-enter the value that you deleted.



8. Use the mean value to determine the values of *a* and *b* in the general form $y = a(b)^x$. Graph the function rule that you think might "fit" the data well. To do so, press [Y=]. Clear out any equations by pressing [CLEAR].

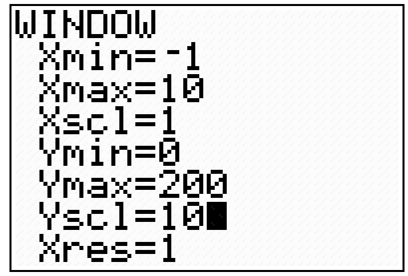


9. Enter the appropriate function rule into Y_1. Press [ENTER]. Press [GRAPH].

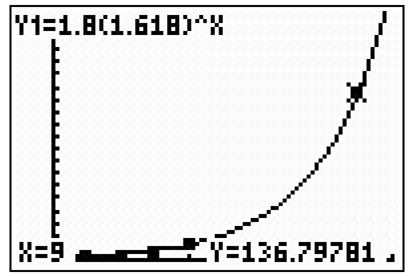


Using the Graph to Make Predictions

1. Press [WINDOW] to adjust the window. Adjust the settings to enlarge the window enough to make predictions.

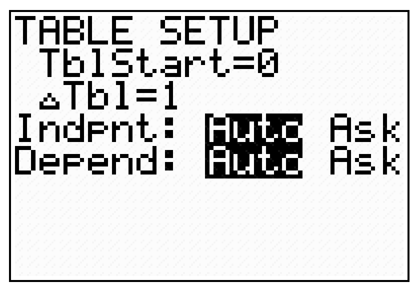


2. Press [GRAPH] then [TRACE]. Press [▲] to select the function then trace to the prediction using the right and left arrow keys, [◄][►].

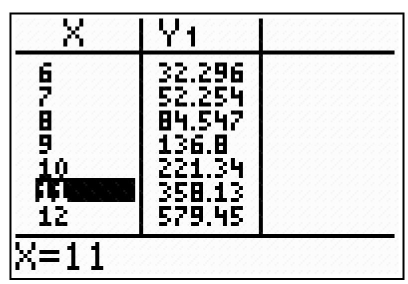


Using the Table to Make Predictions

1. Press [2nd] [WINDOW]. Enter values for TblStart and ΔTbl, the value of the *x* increment.



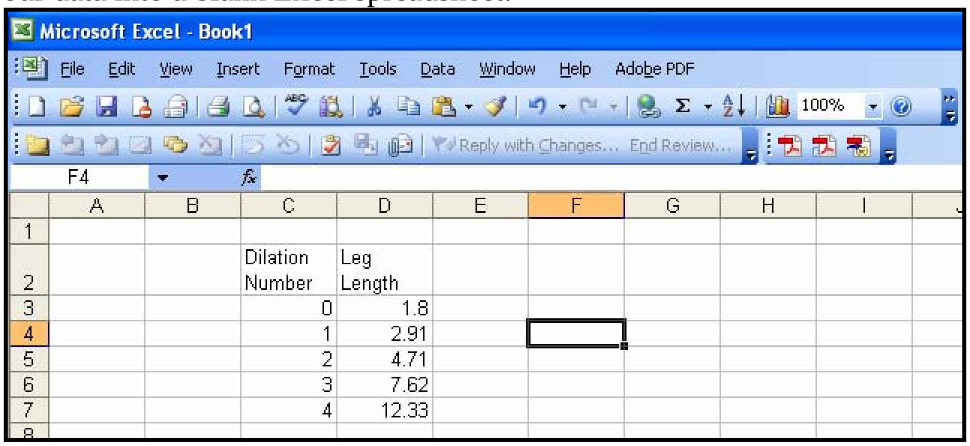
2. Press [2nd] [GRAPH]. Use the up and down arrow keys, [▲] and [▼], to scroll to the desired value.



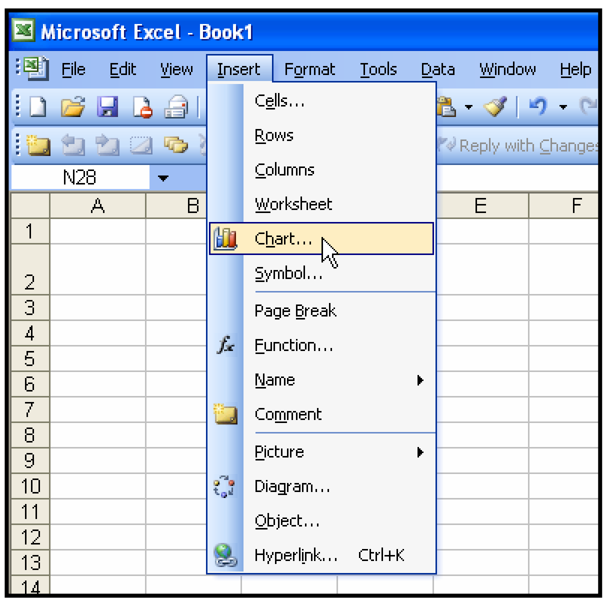
Part 2: Investigating Dilations

Determining a Function Rule for Leg Length vs. Triangle Number Using a Microsoft Excel Spreadsheet

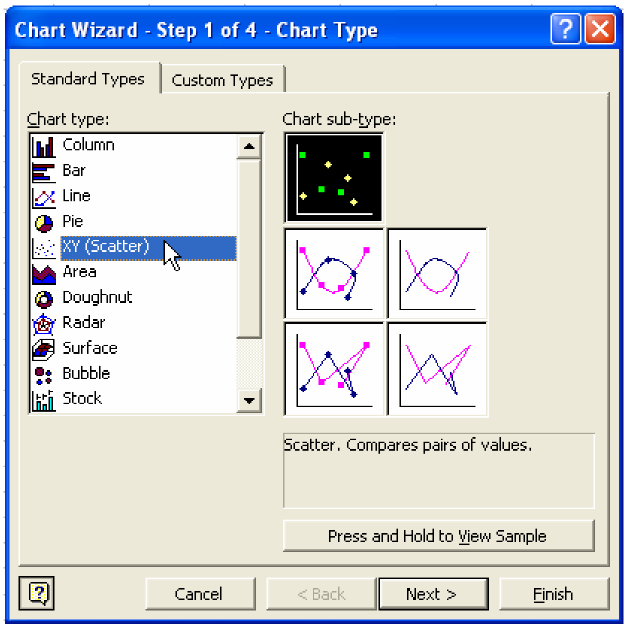
1. Enter your data into a blank Excel spreadsheet.



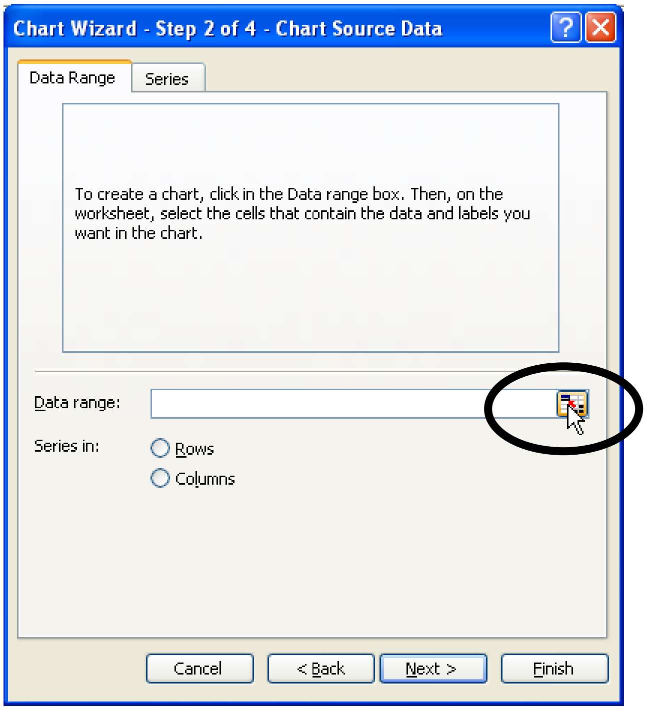
2. Choose **Chart** from the **Insert** menu.



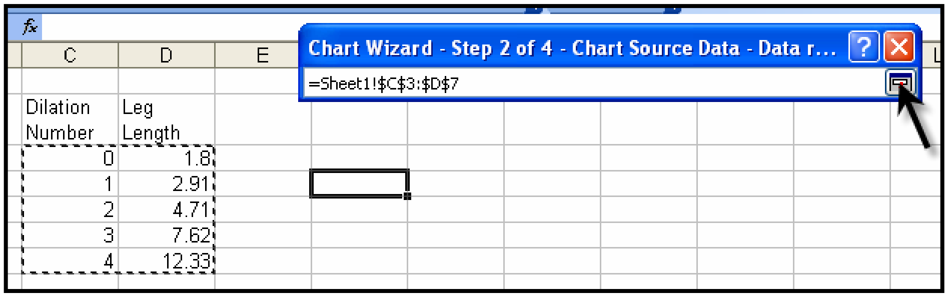
3. Select **XY (Scatter)** from the **Chart Type** selection box then click **Next**.



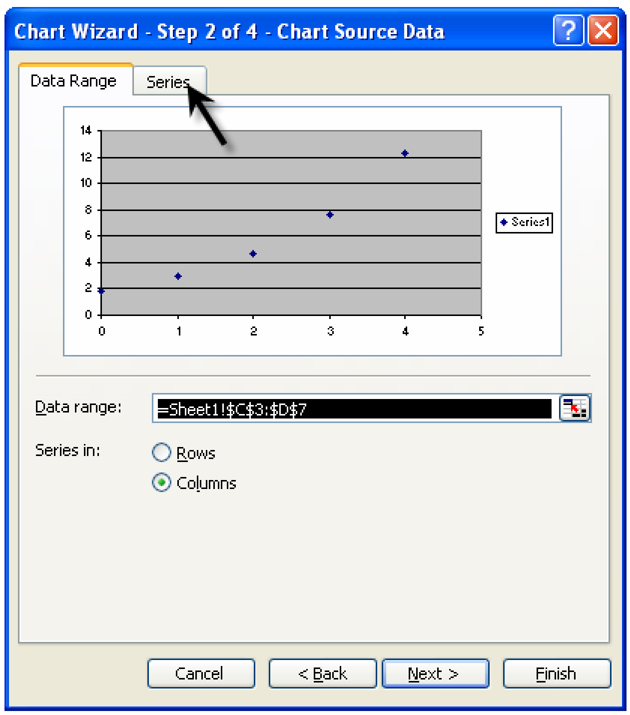
4. To select the Data Range, click the **Collapse Dialog** button next to the **Data Range** text box.



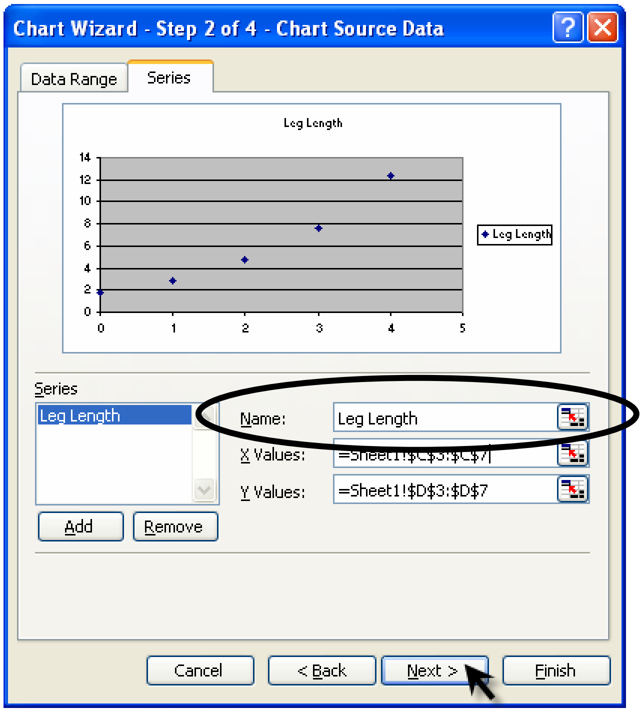
5. Select the cells containing your data then click the **Collapse Dialog** button next to the floating **Chart Source Data** box. You will return to the **Chart Wizard** dialog box.



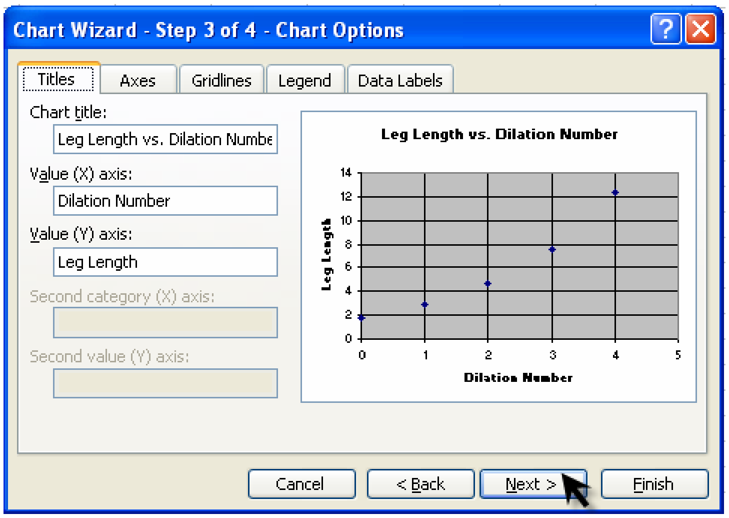
6. Click the **Series** tab to edit the source data features.



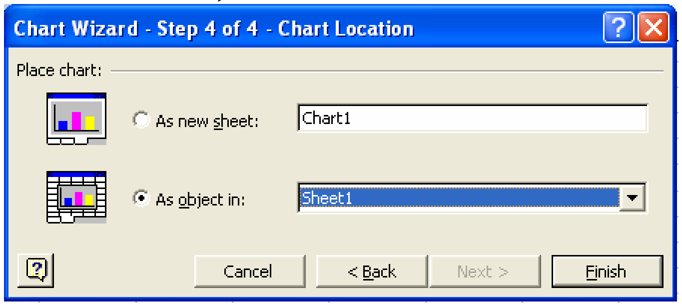
7. Give “Series 1” an appropriate name. Click inside the **Name** text box and type an appropriate name. In this example, we will use “Leg Length.” Click **Next**.

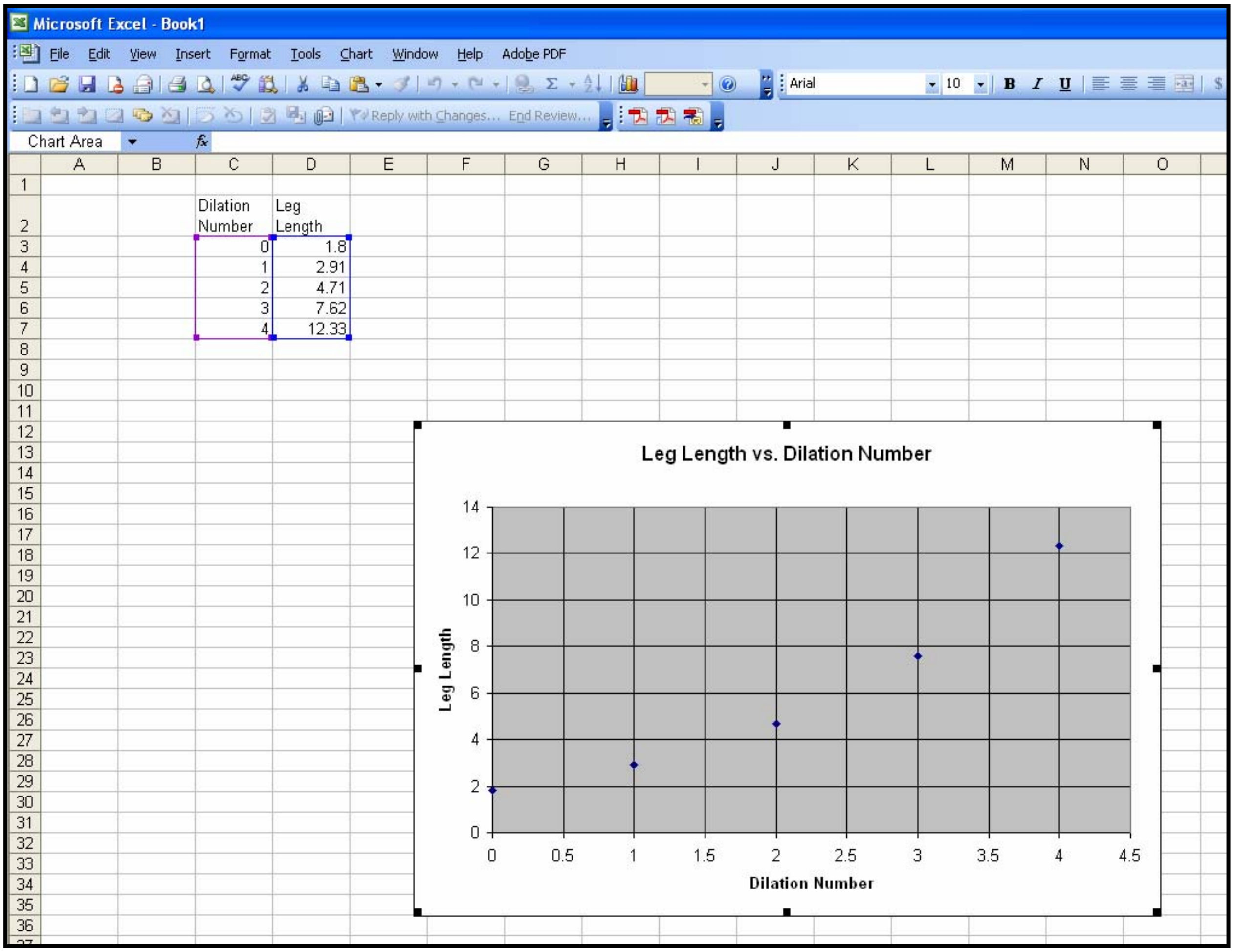


8. At this point you can customize the chart options, including the **Chart title**, **Value (*x*) axis**, and **Value (*y*) axis** labels. Enter the pertinent **Chart Options**, including appropriate labels for the x-axis and y-axis. You can also customize the axes, gridlines, legend, and data labels by clicking on the appropriate tab at the top of the dialog box. Click **Next** when you are ready to continue.

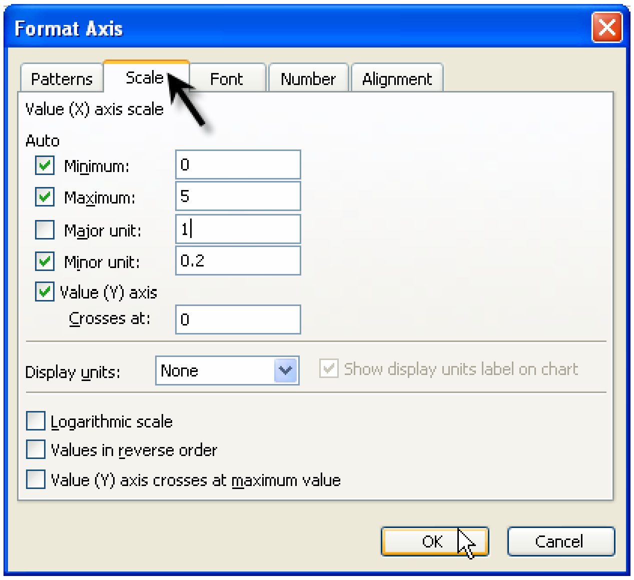


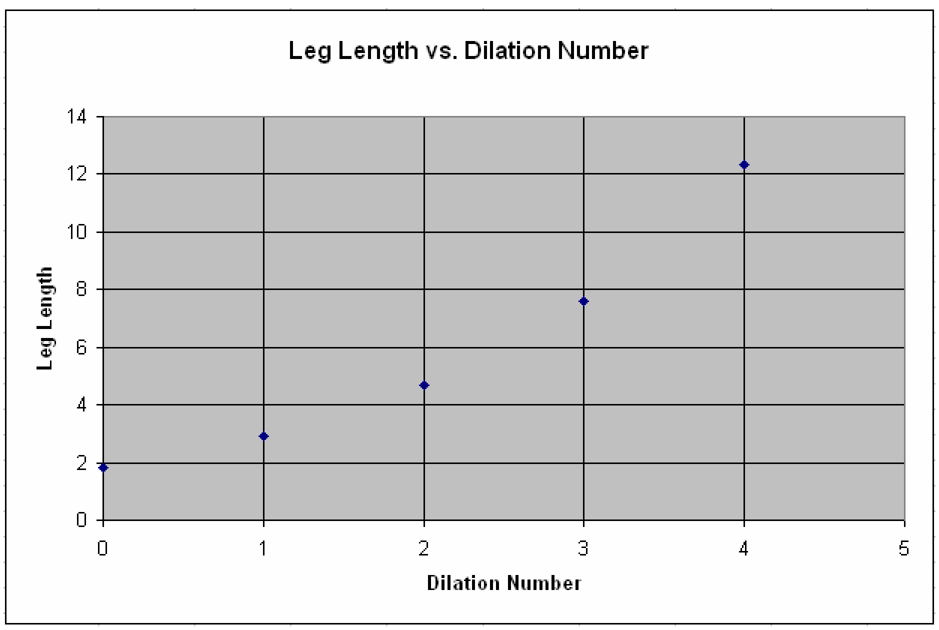
9. Select the location of the new chart, then click Finish.





10. You can customize the features of your chart by double-clicking the part that you wish to change. For example, to change the scale of the *x*-axis, double-click the *x*-axis. The **Format Axis** dialog box will appear. Click on the **Scale** tab, then change the major unit. Click **OK**.

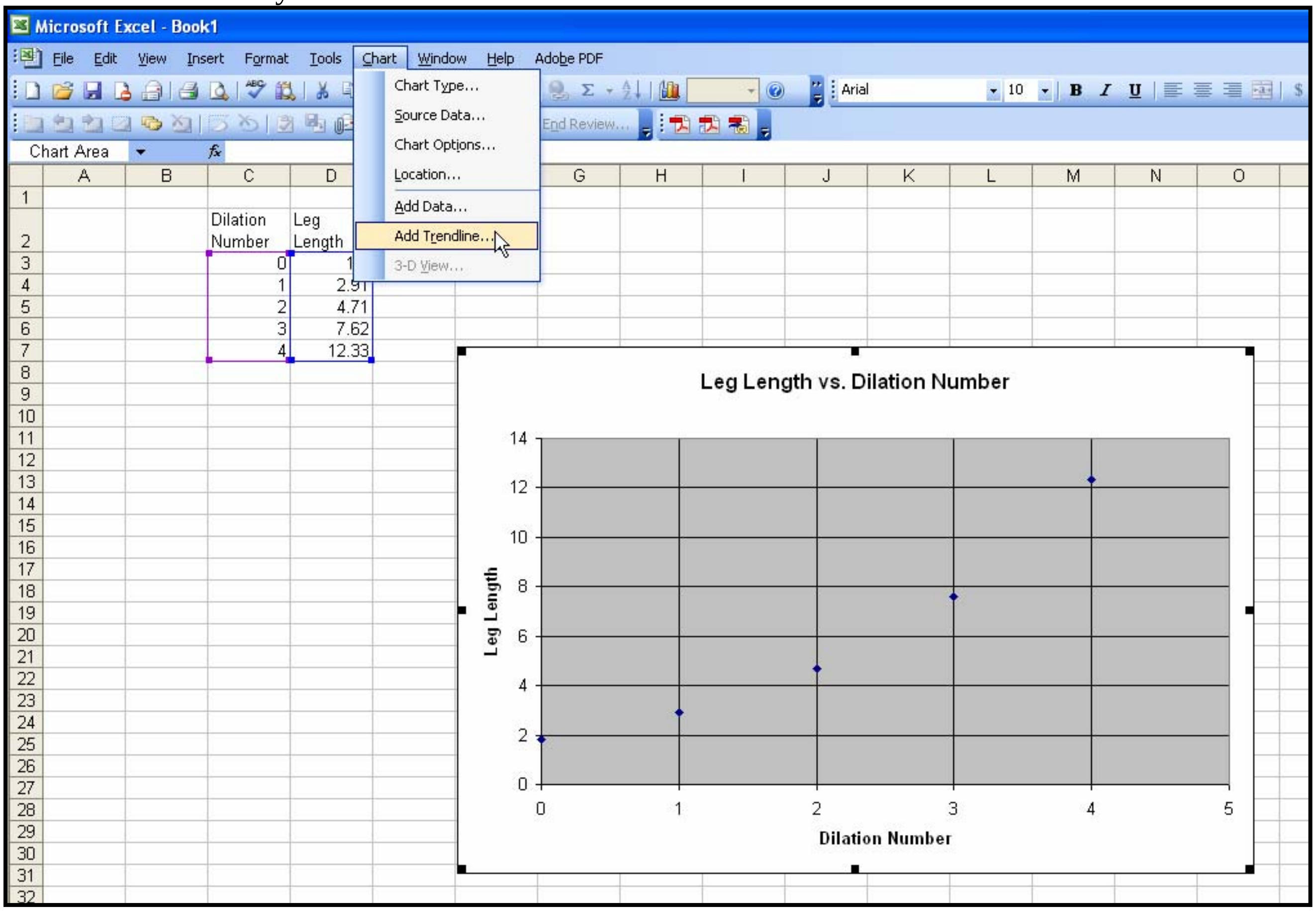




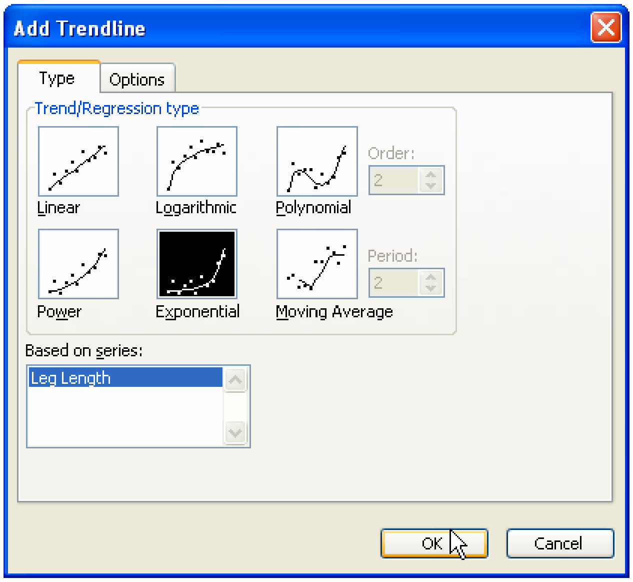
Part 2: Investigating Dilations

Determining a Function Rule for Leg Length vs. Triangle Number Using a Microsoft Excel Spreadsheet

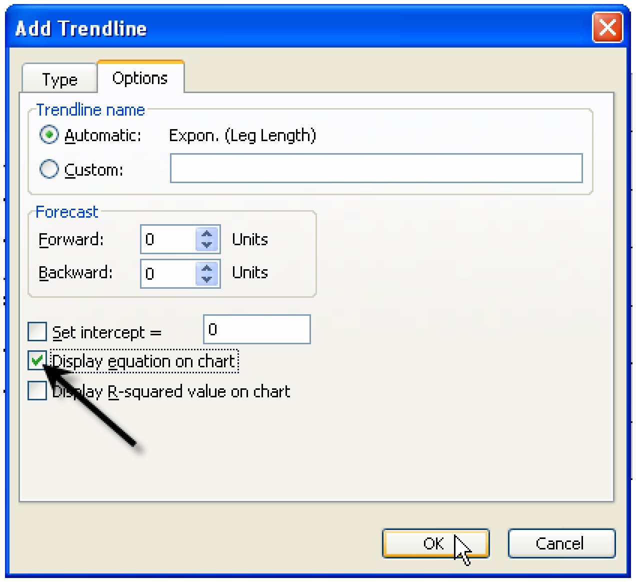
5. Click to select your chart. Choose **Add Trendline** from the **Chart** menu.



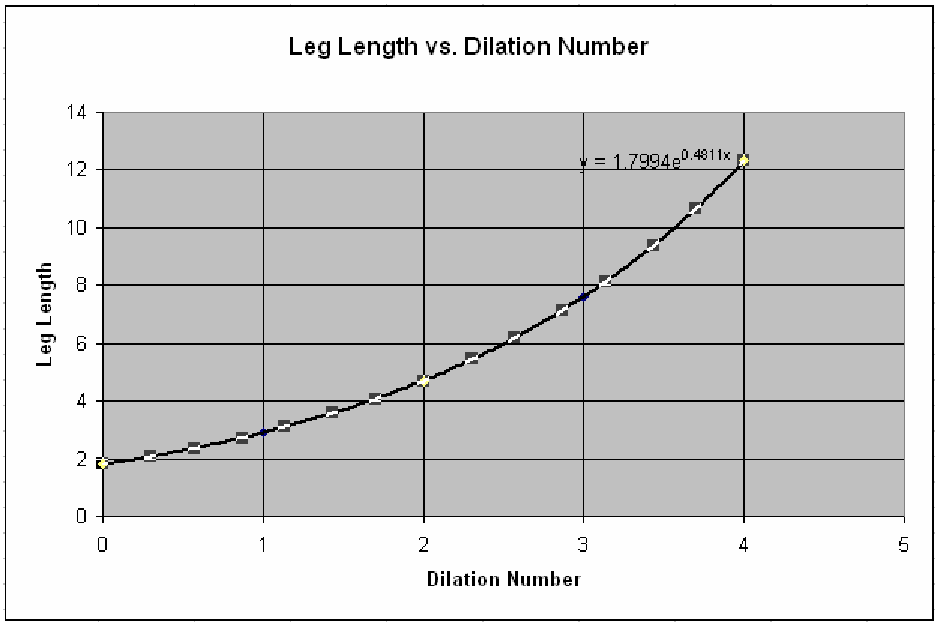
6. The **Add Trendline** dialog box will appear. Click on the **parent function** for the trendline you wish to graph. If you select **Polynomial** or **Moving Average**, be sure to select the order or period, respectively.

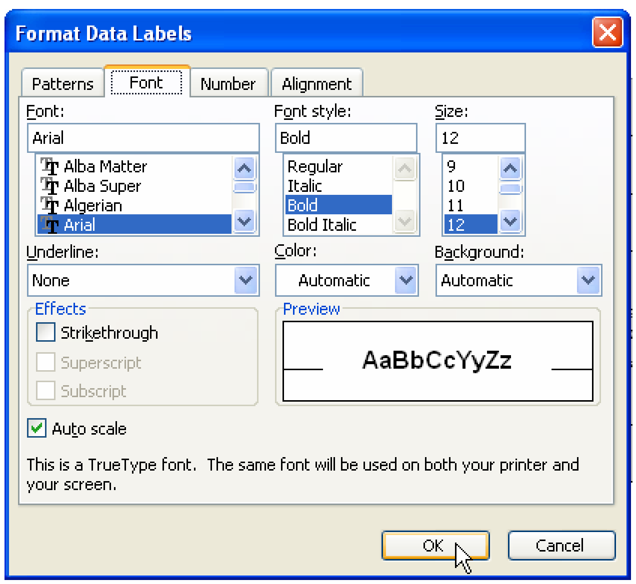


7. Click on the **Options** tab. Click on the **Display equation on chart** check box. Set any other features that you would like to customize related to your trend line. Click **OK**.



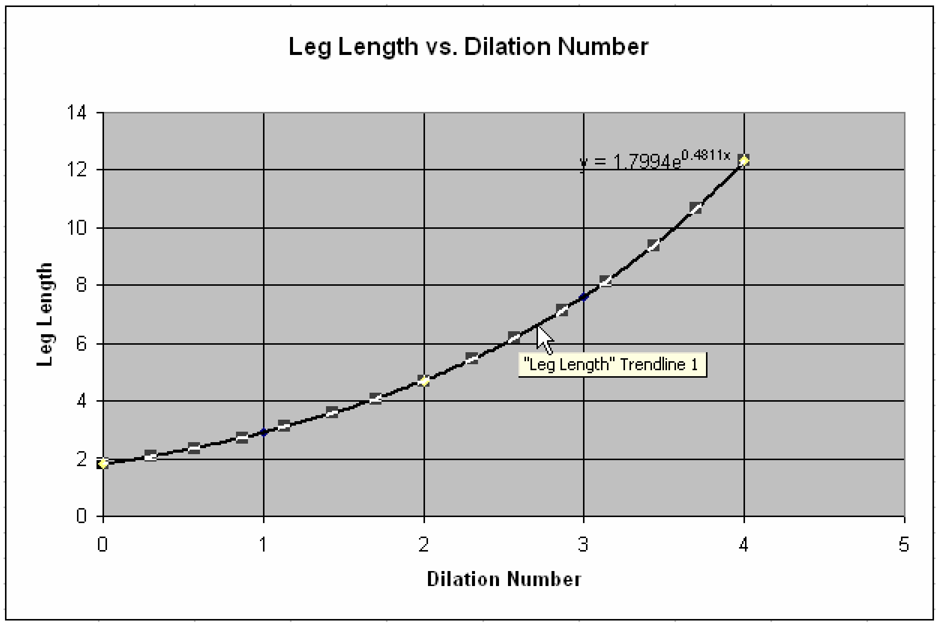
8. Customize the appearance of the equation by double-clicking on the equation. The **Format Data Labels** dialog box will appear. You can change the appearance of the equation, including font, number, and alignment. Click **OK** when you are finished.



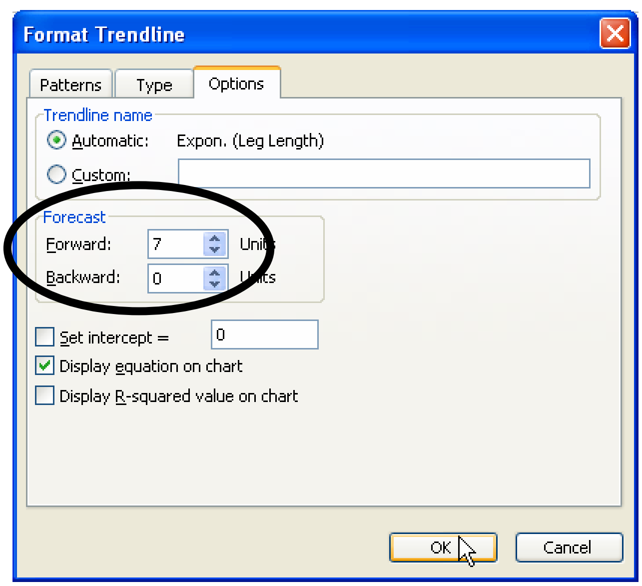


Using the Graph to Make Predictions

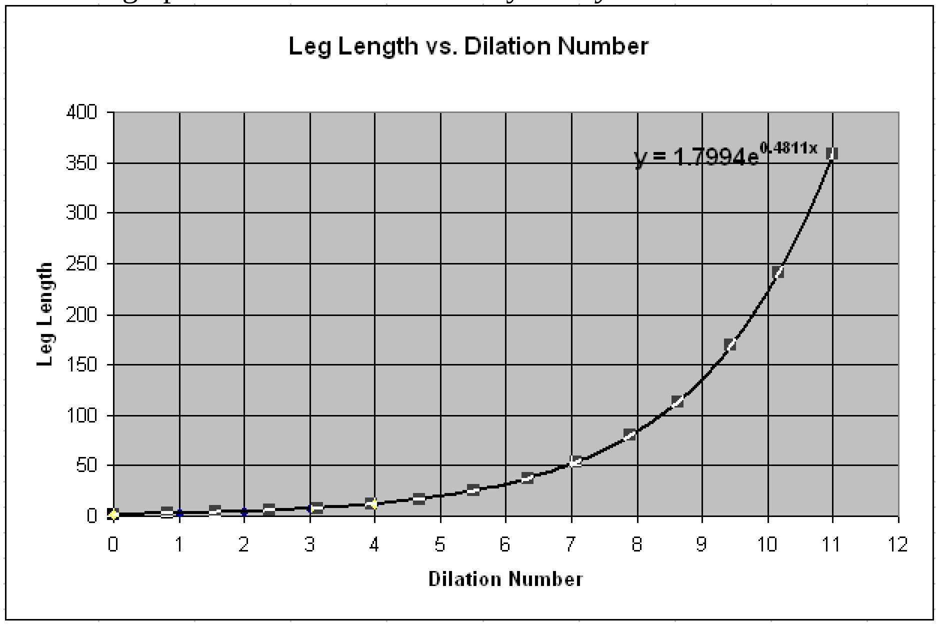
4. Double-click the trendline on your chart. The Format Trendline dialog box will appear.



5. Click the **Options** tab. In the **Forecast** text boxes, enter the number of units that you would like to extend the graph either **Forward** or **Backward** beyond your data set. Click **OK**.



6. Use the extended graph to estimate the necessary *x*- or *y*-value.

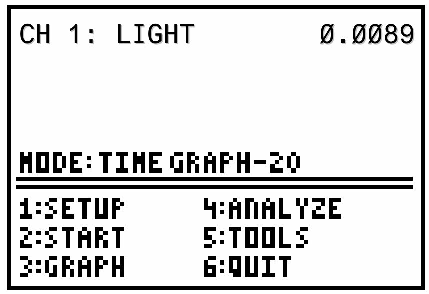


Using the CBL2 and Light Probe to Collect Data

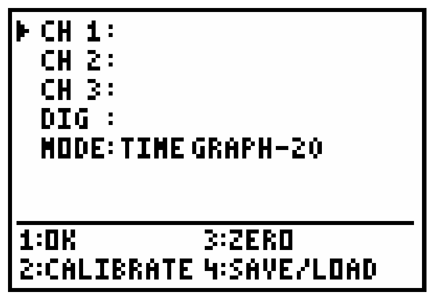
1. Plug the light sensor into a Channel port of your CBL2. Run a data collection program, such as the DataMate App. Press [APPS], then use [▼] to scroll down to DataMate.

   The DataMate program will automatically recognize the light sensor. The number in the top right corner is the reading of light intensity in milliwatts per square centimeter.

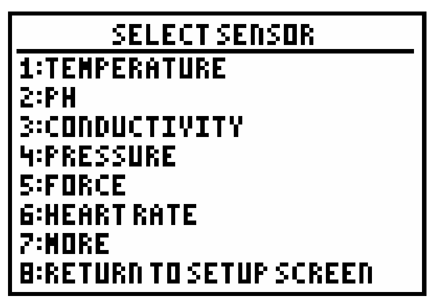
2. If DataMate does not automatically recognize the light sensor, then select option 1: SETUP by pressing [1].

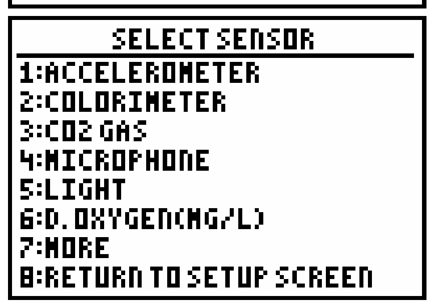


3. Select the Channel port into which you plugged the light sensor. Press [▲] or [▼] so that the arrow is next to the appropriate Channel. Press [ENTER].

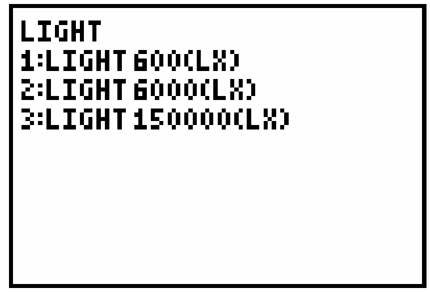


4. Look for the **LIGHT** sensor. If you do not see it on the current screen, select **7: MORE** by pressing [7]. When you see **LIGHT** listed, select **5: LIGHT** by pressing [5].

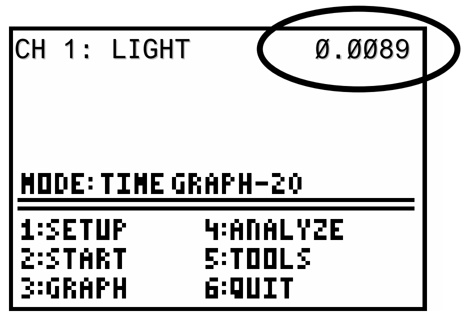




5. Select the light probe that you are using by pressing [1], [2], or [3]. You will be returned to the main screen.



6. Read the light intensity (in milliwatts per square centimeter) by observing the number in the top-right corner of the screen.

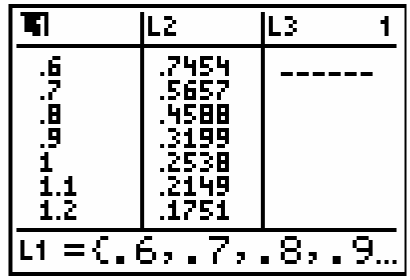


7. To collect the next data point, move the light probe away from the light source, then read the intensity. Continue until you have collected the necessary data.

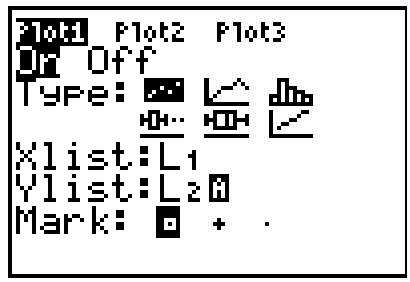
8. Press [6] to return to the home screen.

Generating a Scatterplot Using a Graphing Calculator

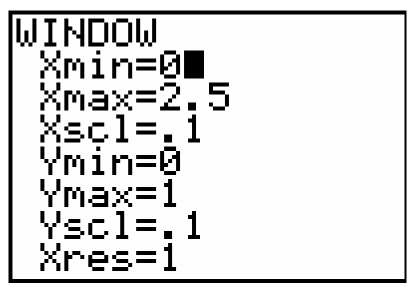
1. Enter data into the [STAT] lists.



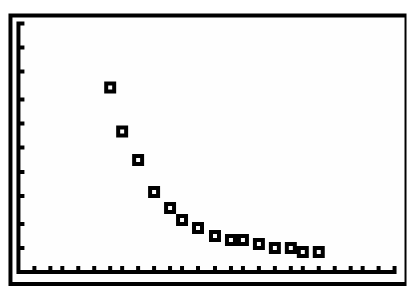
2. Turn on the [STAT PLOT] by pressing [2nd][Y=]. Select the necessary options. In this case, choose a scatterplot with independent variable in [L1] and dependent variable in [L2].



3. Choose an appropriate window by pressing [WINDOW] and specifying the appropriate domain and range. Use [▲][▼] to move up and down the list. Type the desired value then press [ENTER].

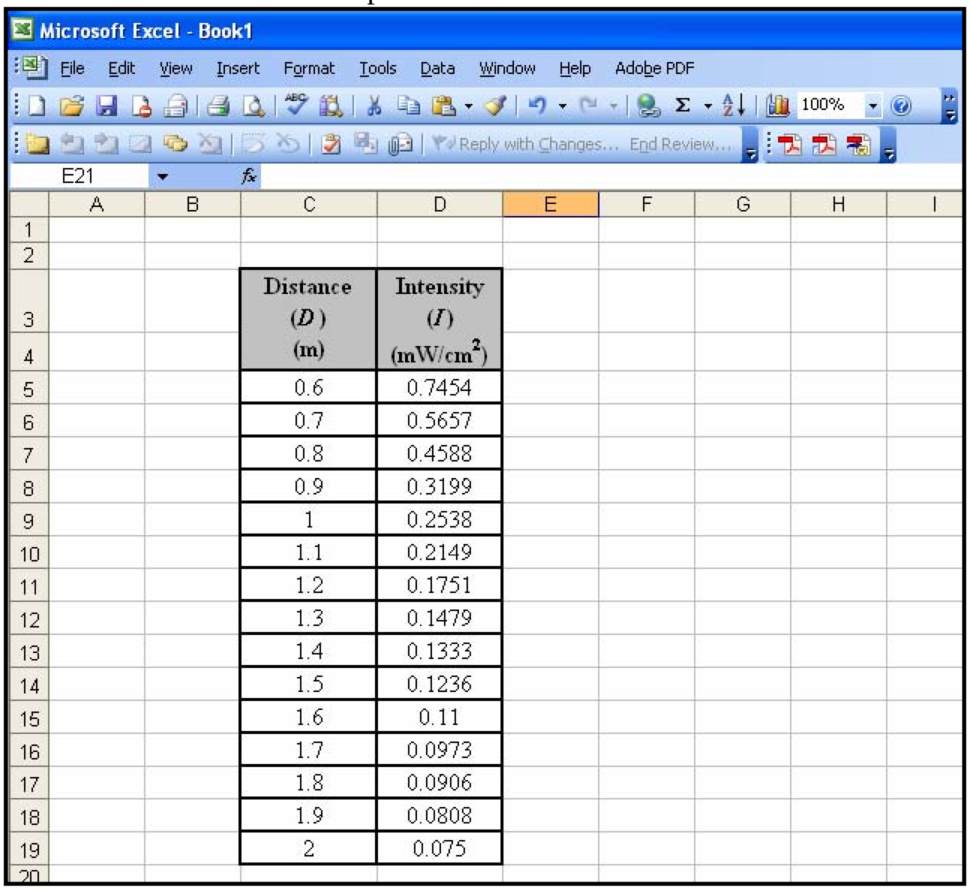


4. To view the graph, select [GRAPH].



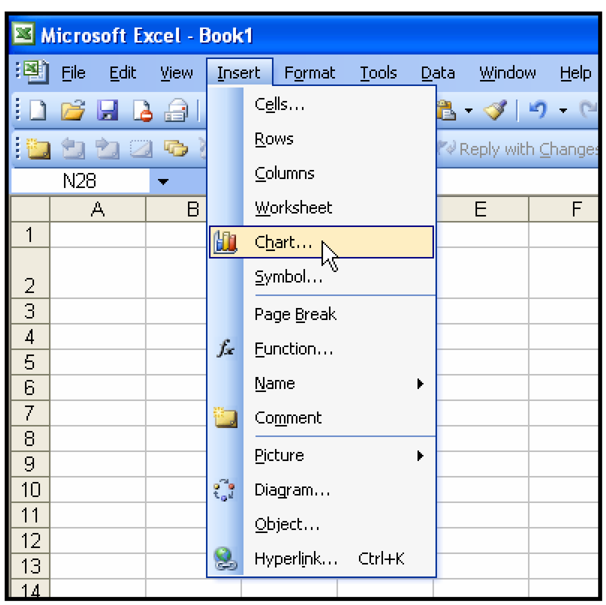
Generating a Scatterplot Using Microsoft Excel

1. Enter your data into a blank Excel spreadsheet.

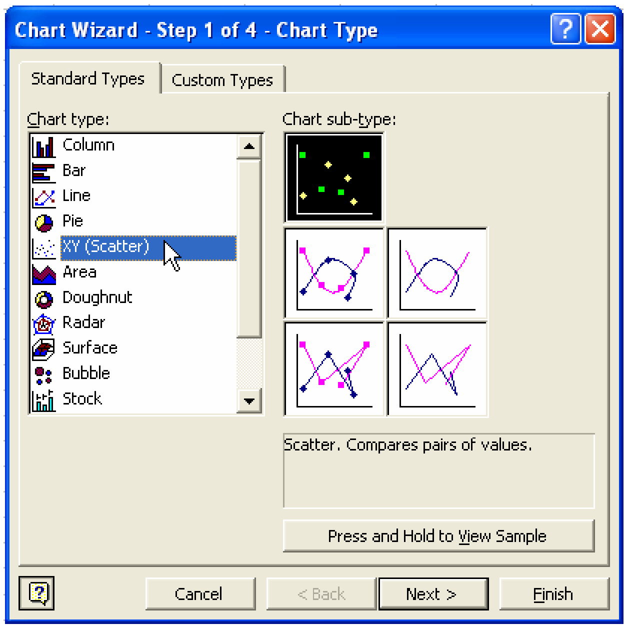


Distance (D) (m)	Intensity (I) (mW/cm^2)
0.6	0.7454
0.7	0.5657
0.8	0.4588
0.9	0.3199
1	0.2538
1.1	0.2149
1.2	0.1751
1.3	0.1479
1.4	0.1333
1.5	0.1236
1.6	0.11
1.7	0.0973
1.8	0.0906
1.9	0.0808
2	0.075

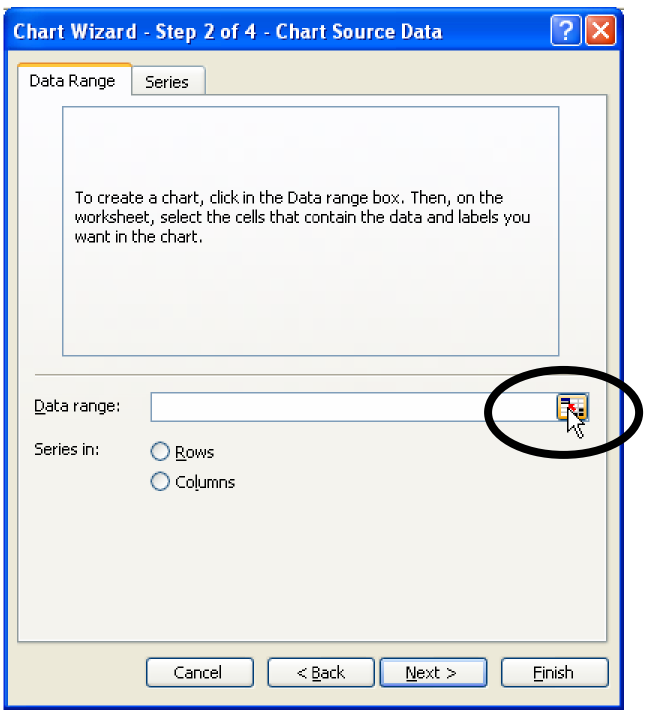
2. Choose **Chart** from the **Insert** menu.



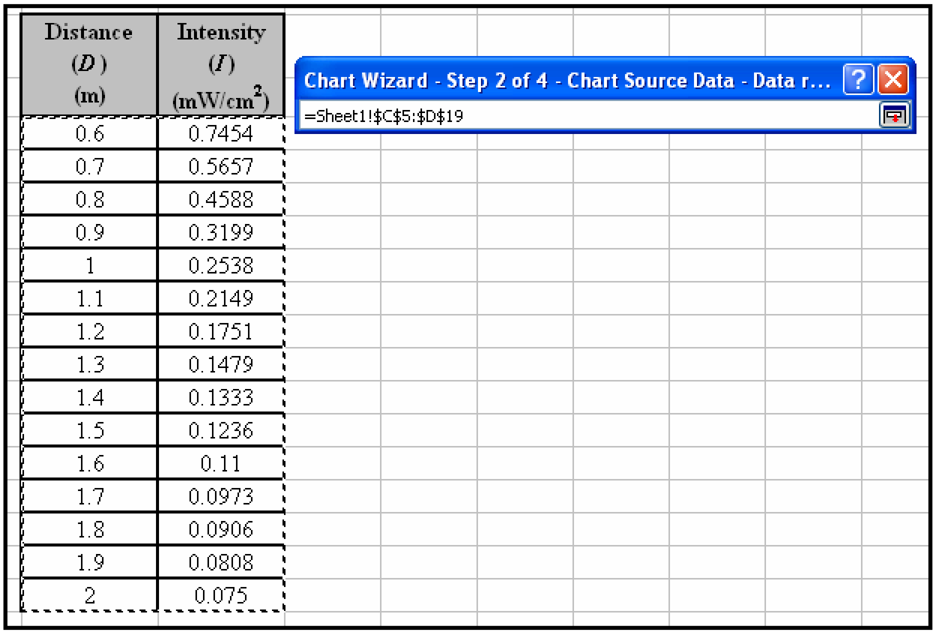
3. Select **XY (Scatter)** from the **Chart Type** selection box then click **Next**.



4. To select the Data Range, click the **Collapse Dialog** button next to the **Data Range** text box.

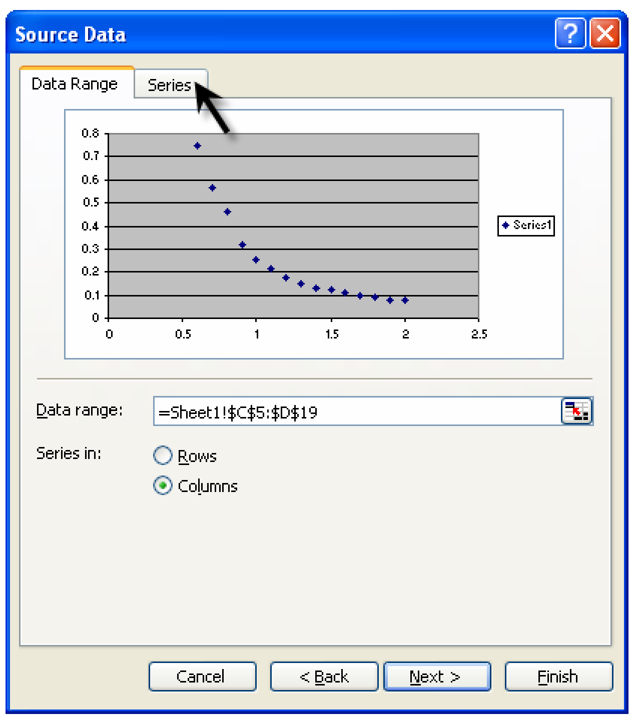


5. Select the cells containing your data then click the **Collapse Dialog** button next to the floating **Chart Source Data** box. You will return to the **Chart Wizard** dialog box.

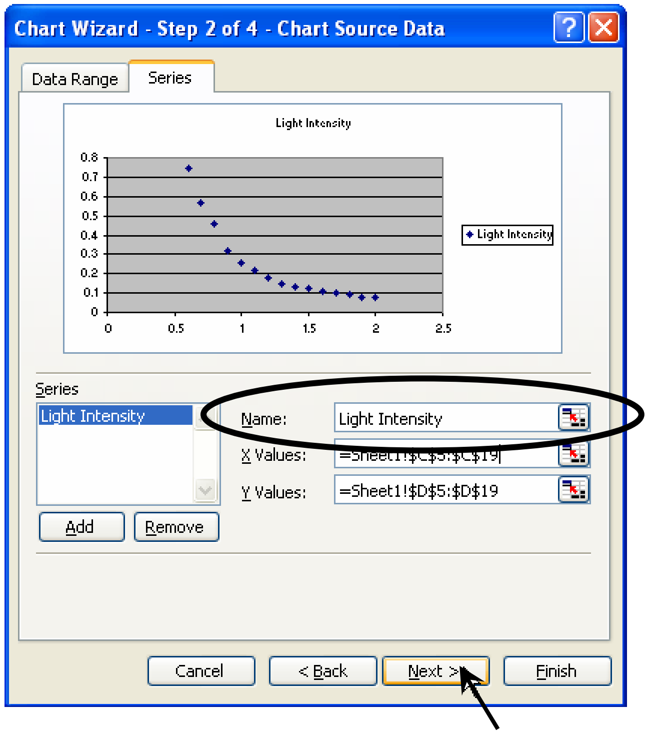


Distance (D) (m)	Intensity (I) (mW/cm^2)
0.6	0.7454
0.7	0.5657
0.8	0.4588
0.9	0.3199
1	0.2538
1.1	0.2149
1.2	0.1751
1.3	0.1479
1.4	0.1333
1.5	0.1236
1.6	0.11
1.7	0.0973
1.8	0.0906
1.9	0.0808
2	0.075

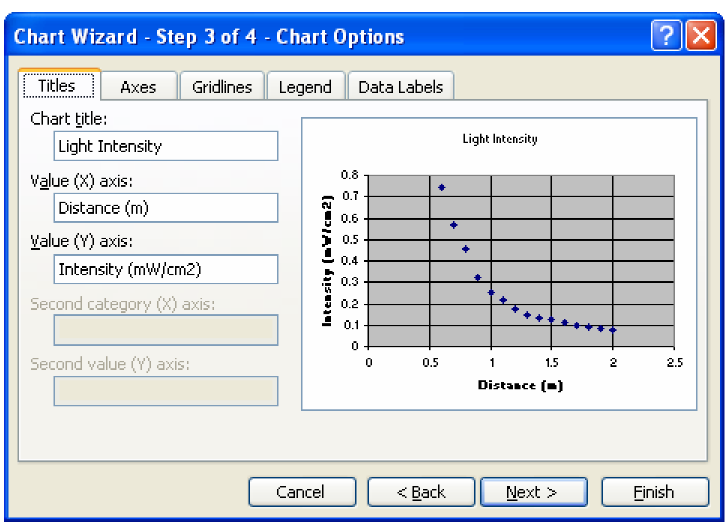
6. Click the **Series** tab to edit the source data features.



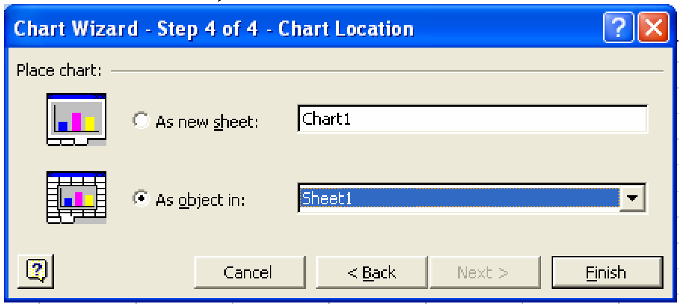
7. Give "Series 1" an appropriate name. Click inside the **Name** text box and type an appropriate name. In this example, we will use "Leg Length." Click **Next**.

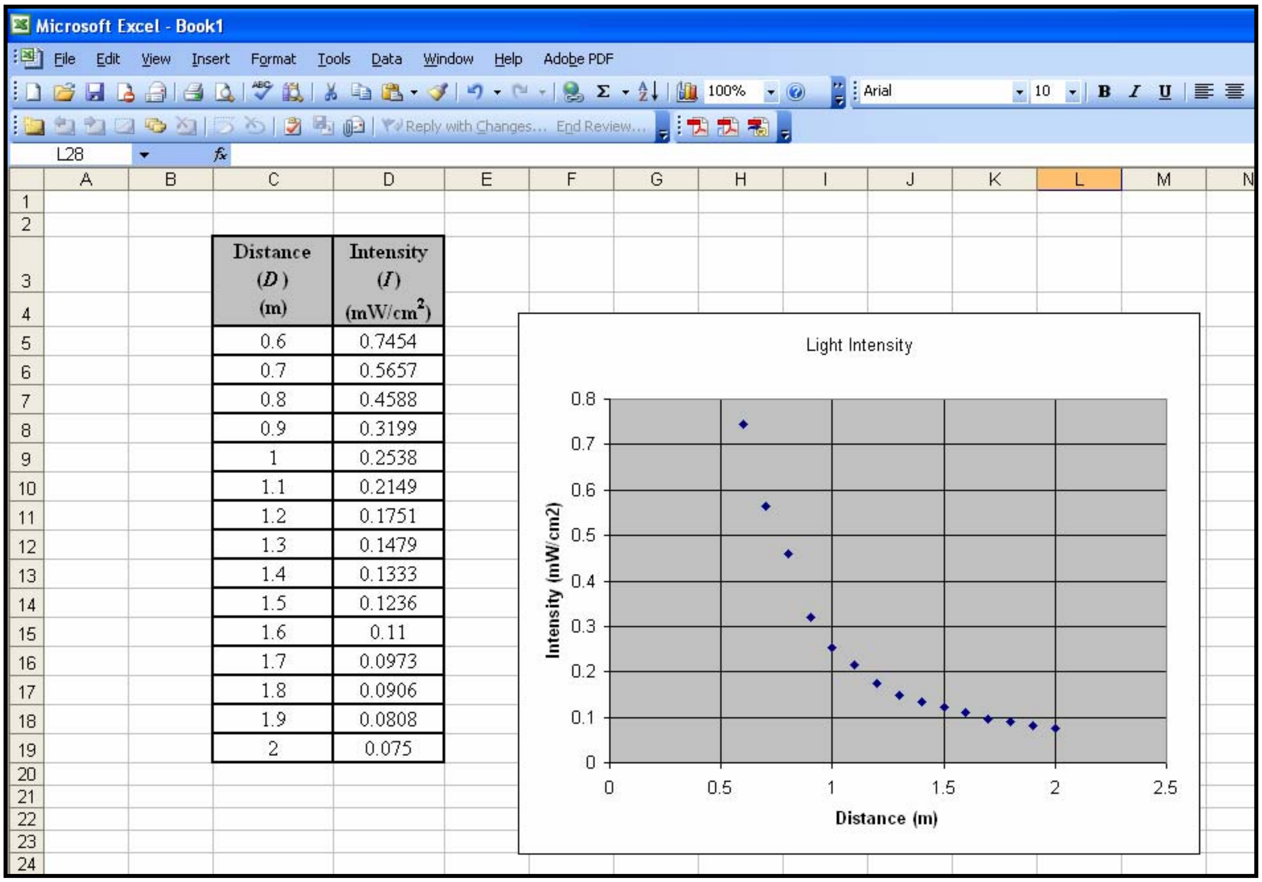


8. At this point you can customize the chart options, including the **Chart title**, **Value (*x*) axis**, and **Value (*y*) axis** labels. Enter the pertinent **Chart Options**, including appropriate labels for the x-axis and y-axis. You can also customize the axes, gridlines, legend, and data labels by clicking on the appropriate tab at the top of the dialog box. Click **Next** when you are ready to continue.



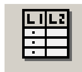
9. Select the location of the new chart, then click **Finish**.

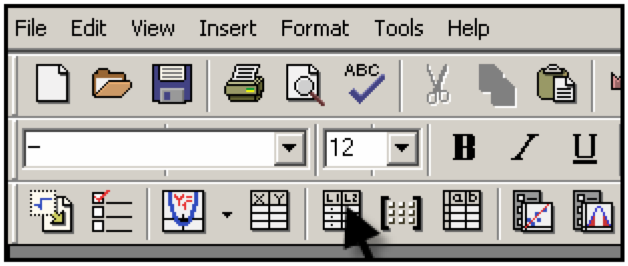




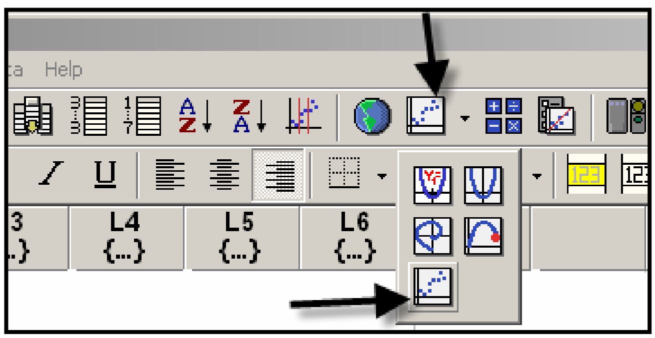
Generating a Scatterplot Using TI-Interactive

1. Open a new TI-Interactive document.

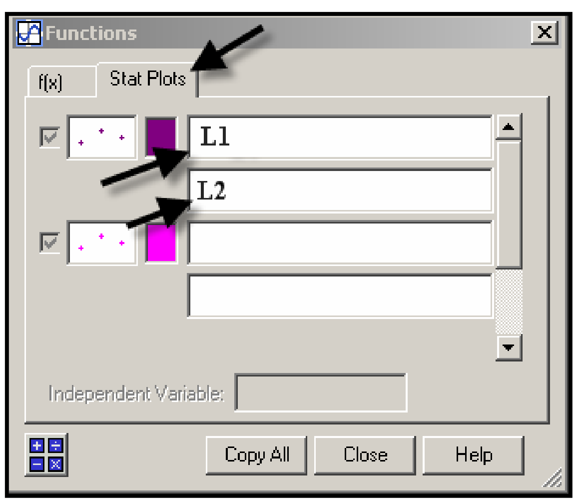
2. Select the list icon  from the scroll bar to activate the **DATA EDITOR**.



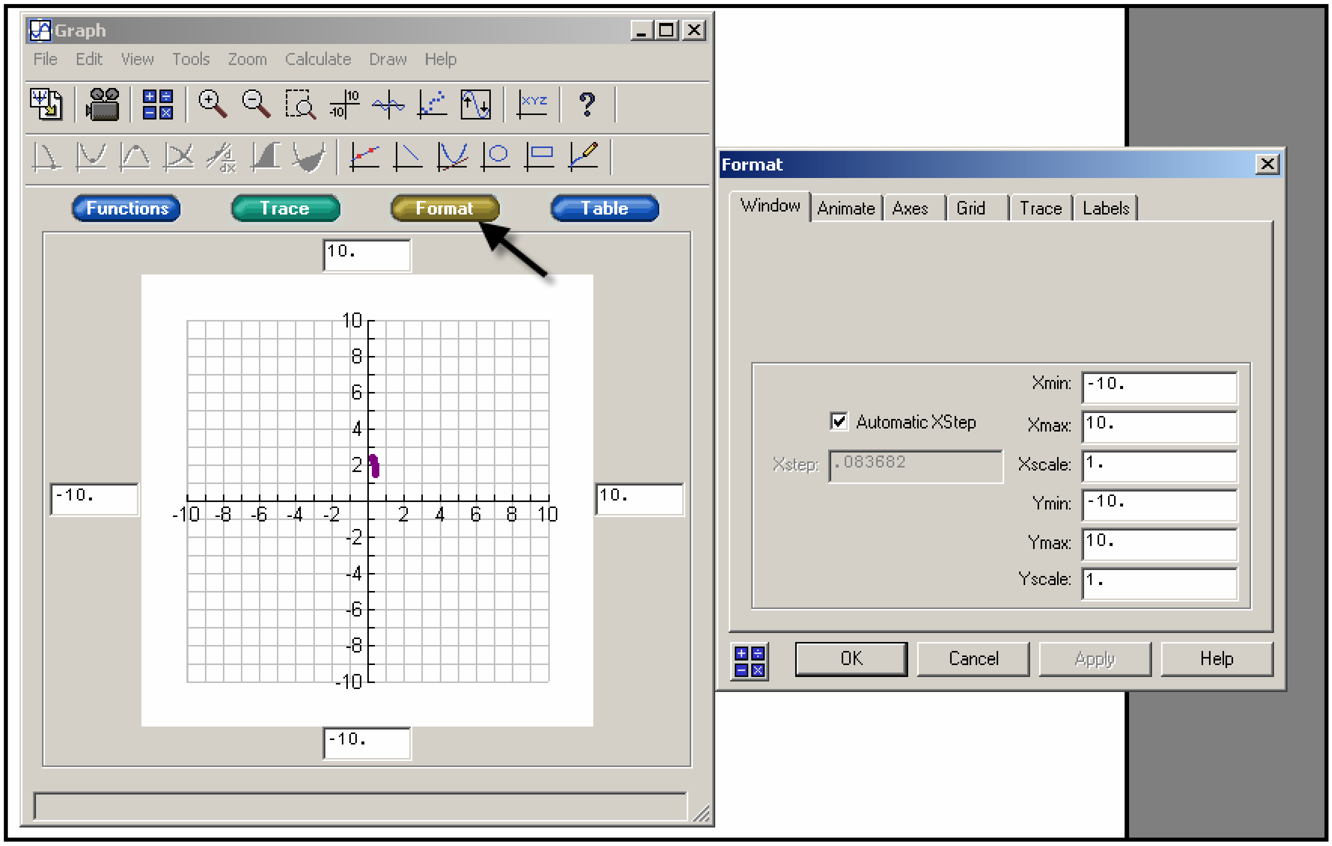
3. Create a scatterplot. Select the scatterplot icon  from the **DATA EDITOR** toolbar and from the drop down menu.



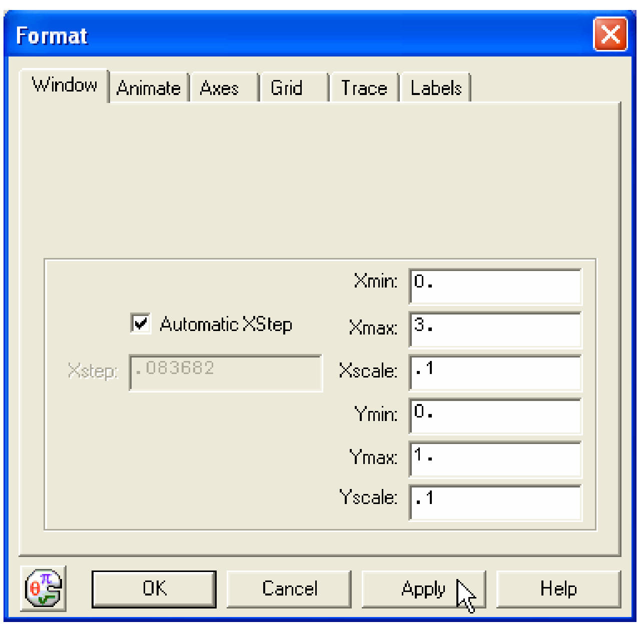
4. Click on the **STAT PLOTS** tab then enter the list names that contain the data, independent variable first and dependent variable second.



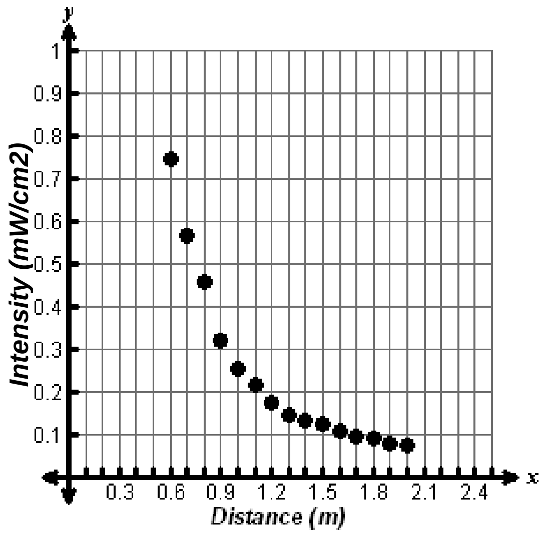
5. Set an appropriate window and label the axes by clicking the **FORMAT** button. In the **Window** tab, enter the appropriate domain and range for the function.



6. After entering the *Xmin*, *Xmax*, *Xscale*, *Ymin*, *Ymax*, and *Yscale*, click the **APPLY** button.



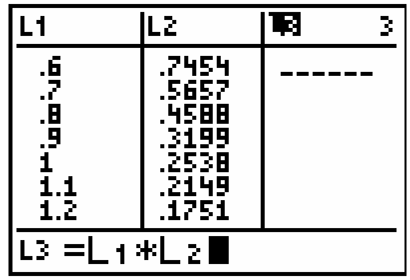
7. The scatterplot should be displayed with the specified domain and range.



Determining a Function Rule Using a Graphing Calculator

1. The graph appears to be an inverse variation function, $y = \frac{k}{x}$, so multiply *xy* to find *k*, the constant of variation.

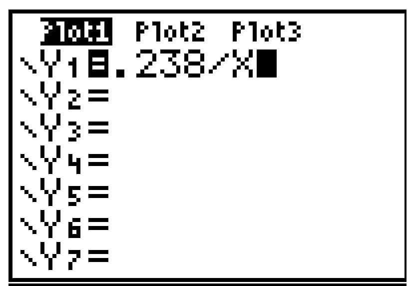
   Go to the List Editor by pressing [STAT][ENTER]. Use [▼][▲] to select the List 3 header. Enter the formula [L3] = [L1] [L2] by pressing [2nd][1][×][2nd][2]. Press [ENTER].

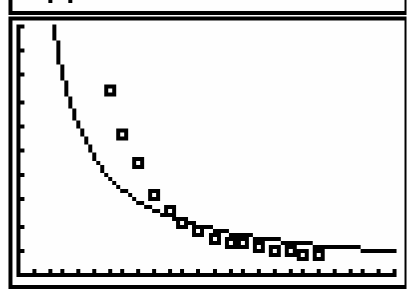


2. Find the average value of List 3 by returning to the home screen and using List operations. Press [2nd][MODE]. Press [2nd][STAT][▶][▶][3]. Enter [L3] by pressing [2nd][3], then press [ENTER].

3. Substitute this value of *k* into the parent function and verify using a graph.

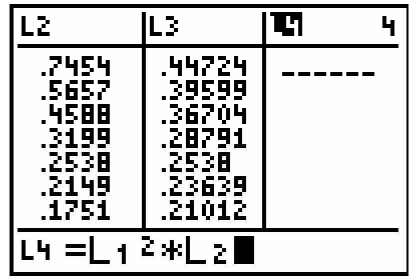
   Press [Y=] then enter the function. Press [GRAPH] to view the graph.



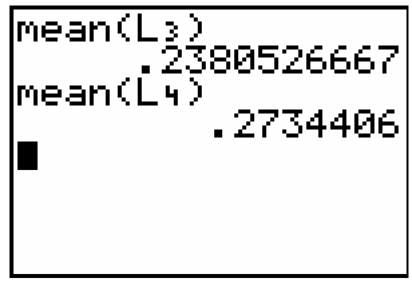


4. This function is not a good fit. Try inverse-square variation, $y = \dfrac{k}{x^2}$. Multiply x^2y in order to find an approximate value for *k*, the constant of variation.

   Go to the List Editor by pressing [STAT][ENTER]. Use [▼][▲] to select the List 4 header. Enter the formula [L4] = [L1]² [L2] by pressing [2nd][1][x²][×][2nd][2]. Press [ENTER].

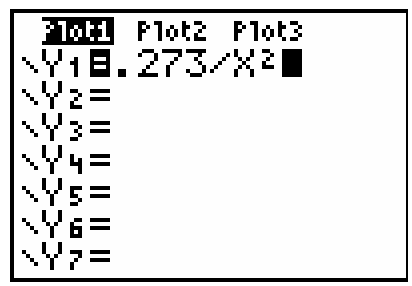


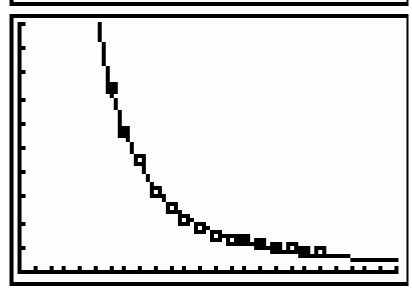
5. Find the average value of List 4 by returning to the home screen and using List operations. Press [2nd][MODE]. Press [2nd][STAT][▶][▶][3]. Enter [L4] by pressing [2nd][4], then press [ENTER].



6. Substitute this value of *k* into the parent function and verify using a graph.

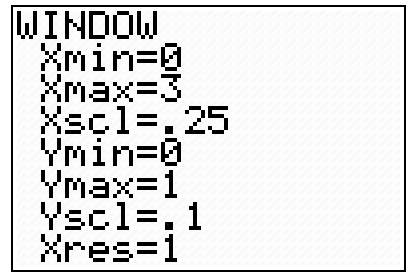
   Press [Y=], then enter the function. Press [GRAPH] to view the graph.



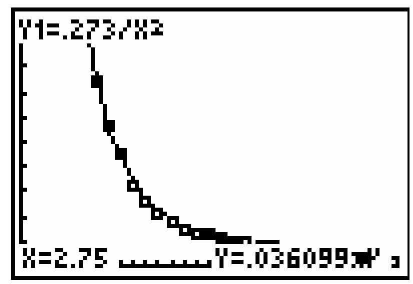


Using the Graph to Make Predictions

1. Press [WINDOW] to enlarge the window. Adjust the settings to make the window large enough to predict with.

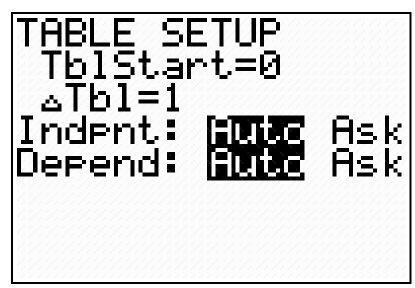


2. Press [GRAPH] then [TRACE]. Press [▲] to select the function then trace to the prediction using the right and left arrow keys, [◄][►].

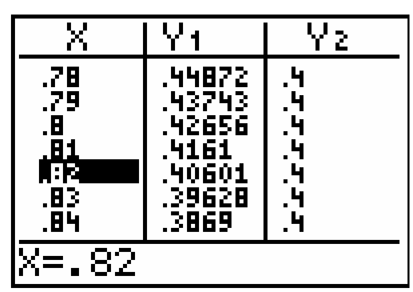


Using the Table to Make Predictions

1. Press [2nd] [WINDOW]. Enter values for TblStart and ΔTbl, the value of the *x* increment.

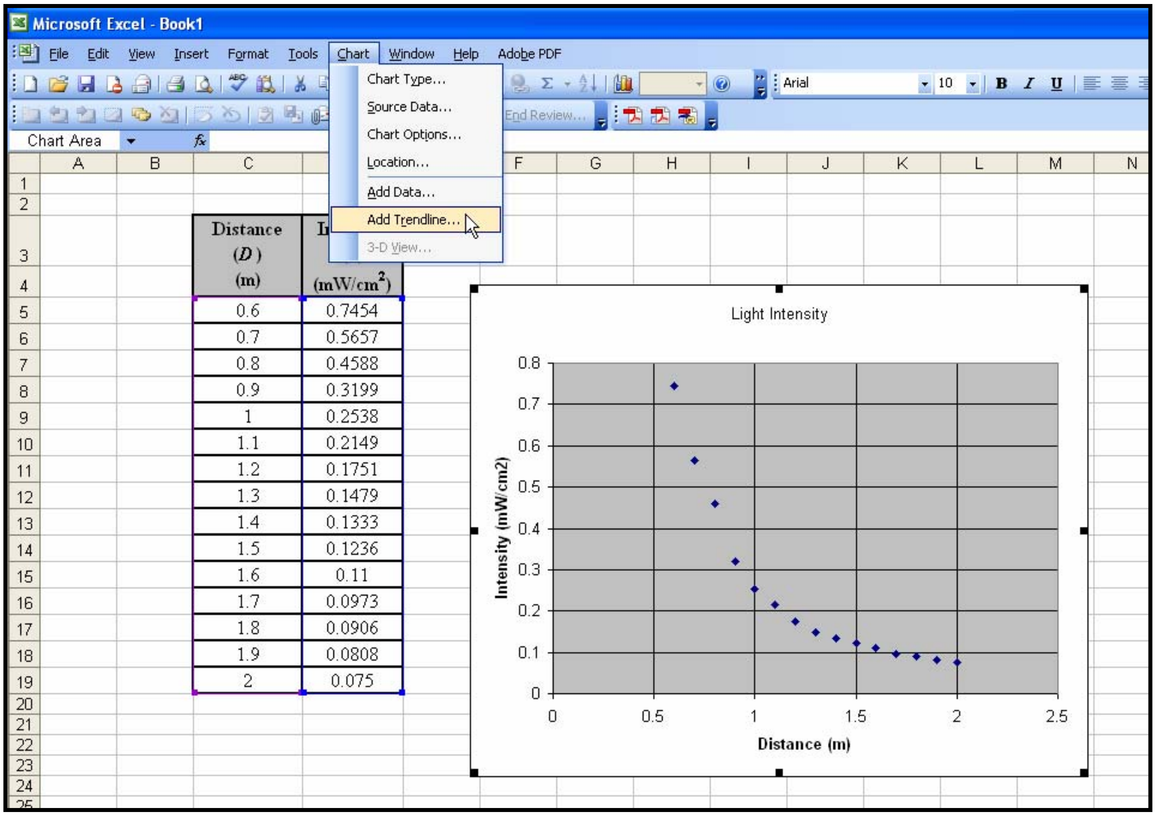


2. Press [2nd] [GRAPH]. Use the up and down arrow keys, [▲] and [▼], to scroll to the desired value.

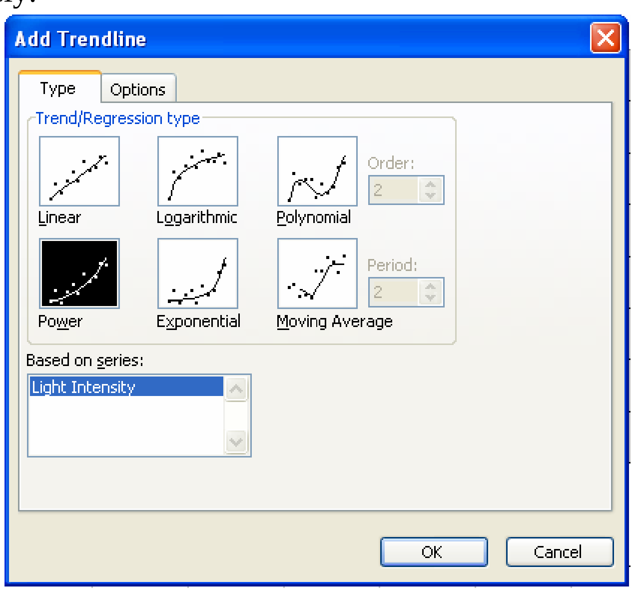


Determining a Function Rule Using Microsoft Excel

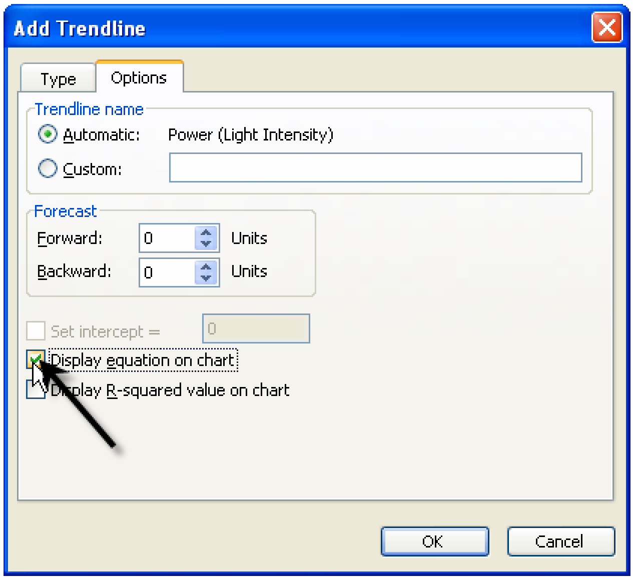
1. Click to select your chart. Choose **Add Trendline** from the **Chart** menu.



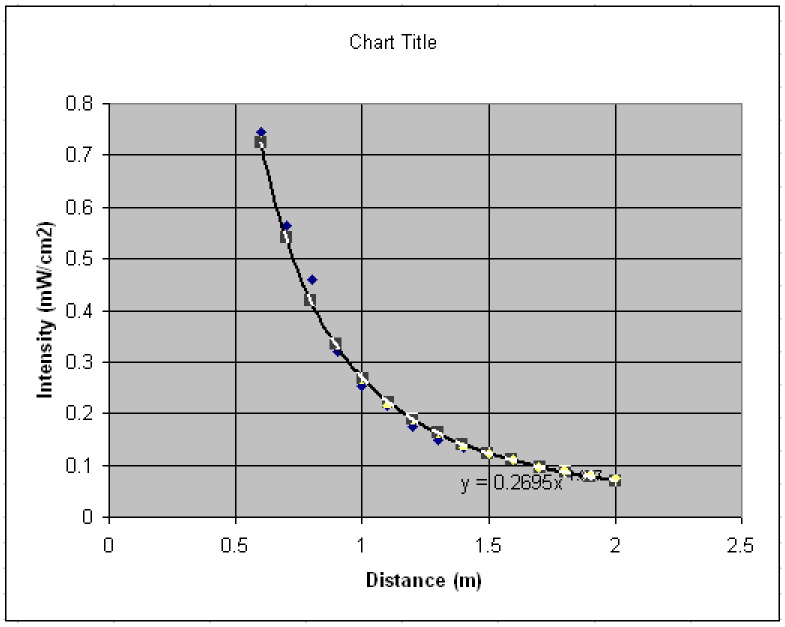
2. The **Add Trendline** dialog box will appear. Click on the **parent function** for the trendline you wish to graph. If you select **Polynomial** or **Moving Average**, be sure to select the order or period, respectively.

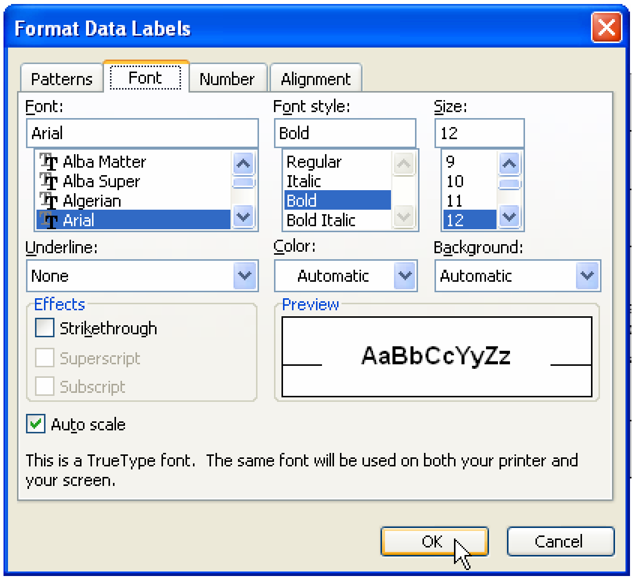


3. Click on the **Options** tab. Click on the **Display equation on chart** check box. Set any other features that you would like to customize related to your trend line. Click **OK**.



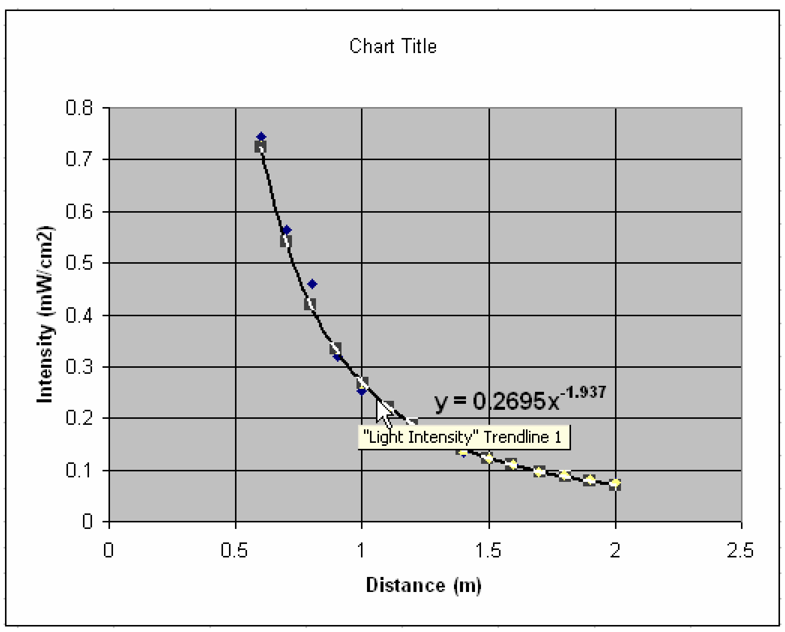
4. Customize the appearance of the equation by double-clicking on the equation. The **Format Data Labels** dialog box will appear. You can change the appearance of the equation, including font, number, and alignment. Click **OK** when you are finished.



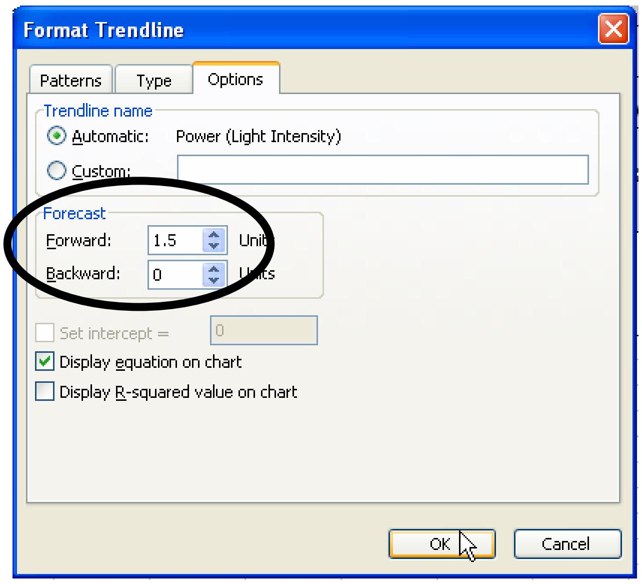


Using the Graph to Make Predictions

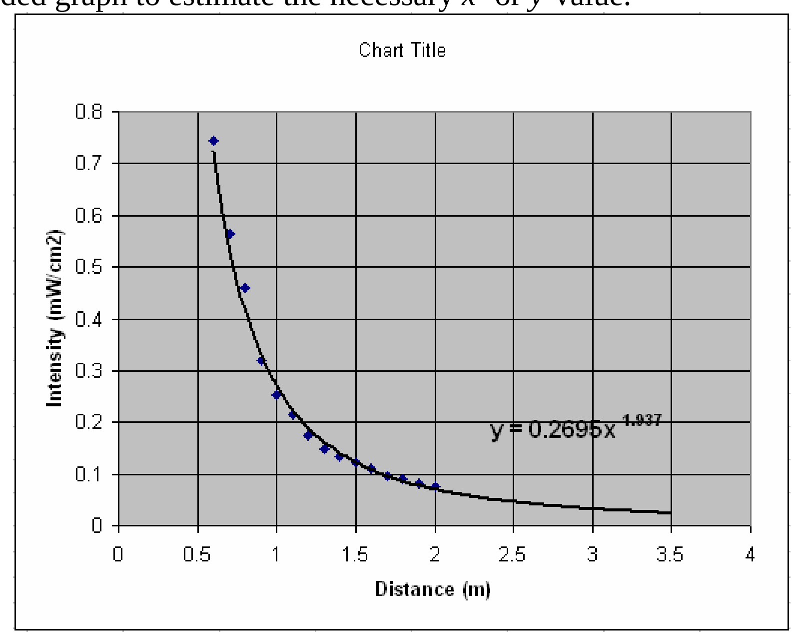
1. Double-click the trendline on your chart. The Format Trendline dialog box will appear.



2. Click the **Options** tab. In the **Forecast** text boxes, enter the number of units that you would like to extend the graph either **Forward** or **Backward** beyond your data set. Click **OK**.

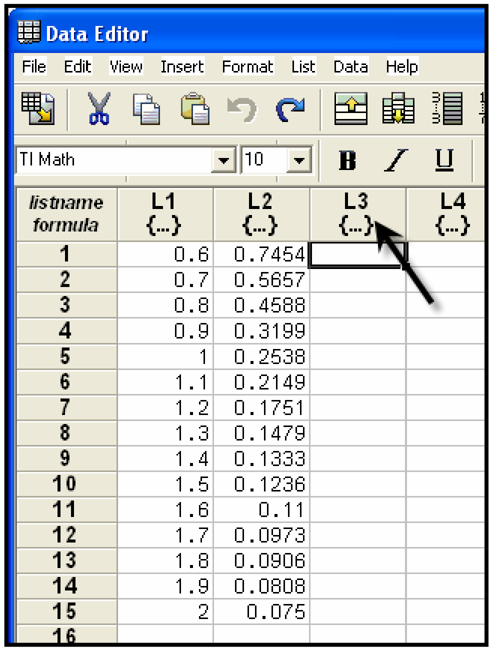


3. Use the extended graph to estimate the necessary *x*- or *y*-value.



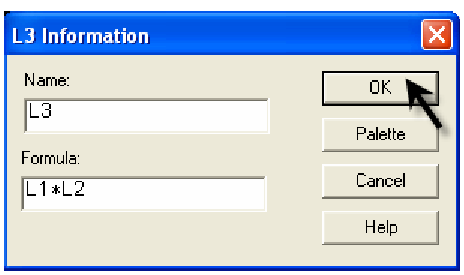
Determining a Function Rule Using TI-Interactive

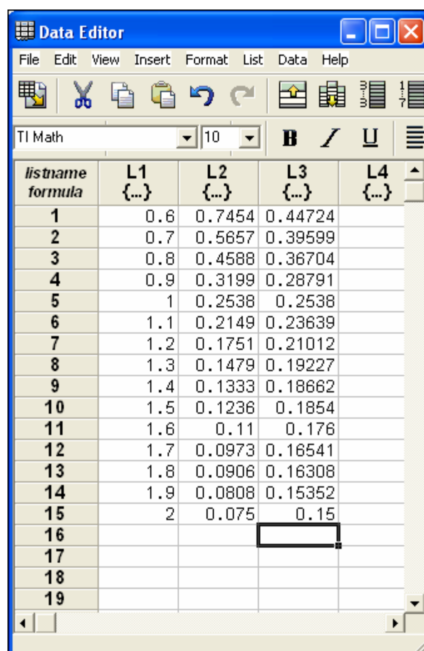
1. The graph appears to be an inverse variation function, $y = \frac{k}{x}$, so multiply *xy* to find *k*, the constant of variation then find the average value. In the **Data Editor**, click the **Formula** tab under the List 3 header.



listname formula	L1 {...}	L2 {...}	L3 {...}	L4 {...}
1	0.6	0.7454		
2	0.7	0.5657		
3	0.8	0.4588		
4	0.9	0.3199		
5	1	0.2538		
6	1.1	0.2149		
7	1.2	0.1751		
8	1.3	0.1479		
9	1.4	0.1333		
10	1.5	0.1236		
11	1.6	0.11		
12	1.7	0.0973		
13	1.8	0.0906		
14	1.9	0.0808		
15	2	0.075		
16				

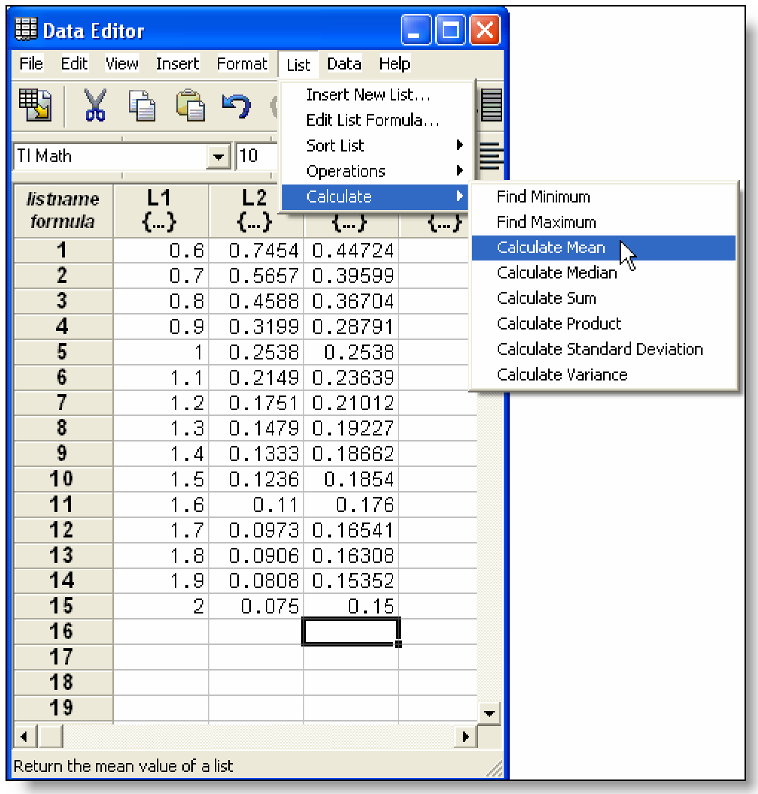
2. Enter the formula **L1*L2** inside the **Formula**: text box. Click **OK**.



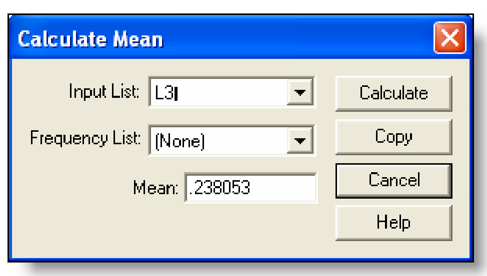


listname formula	L1 {...}	L2 {...}	L3 {...}	L4 {...}
1	0.6	0.7454	0.44724	
2	0.7	0.5657	0.39599	
3	0.8	0.4588	0.36704	
4	0.9	0.3199	0.28791	
5	1	0.2538	0.2538	
6	1.1	0.2149	0.23639	
7	1.2	0.1751	0.21012	
8	1.3	0.1479	0.19227	
9	1.4	0.1333	0.18662	
10	1.5	0.1236	0.1854	
11	1.6	0.11	0.176	
12	1.7	0.0973	0.16541	
13	1.8	0.0906	0.16308	
14	1.9	0.0808	0.15352	
15	2	0.075	0.15	
16				
17				
18				
19				

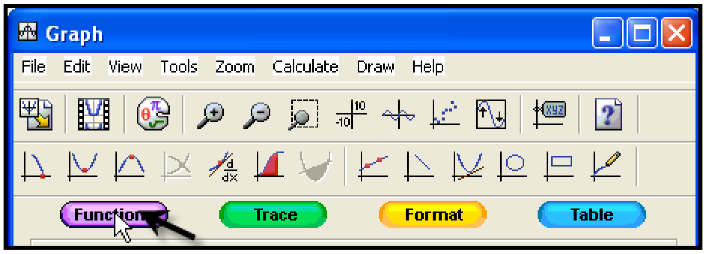
3. From the **List** menu, choose **Calculate**, then choose **Calculate Mean**.



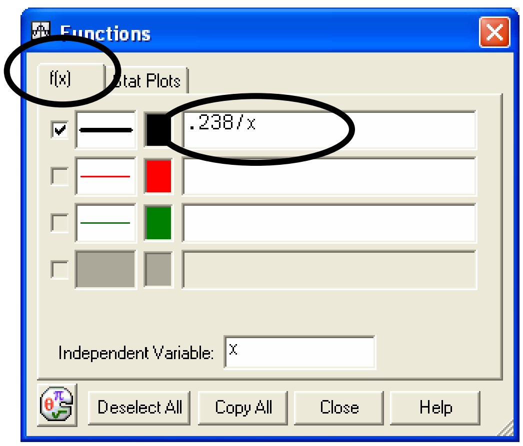
4. From the **Input List** drop-list box, choose **L3**. Click **Calculate**.

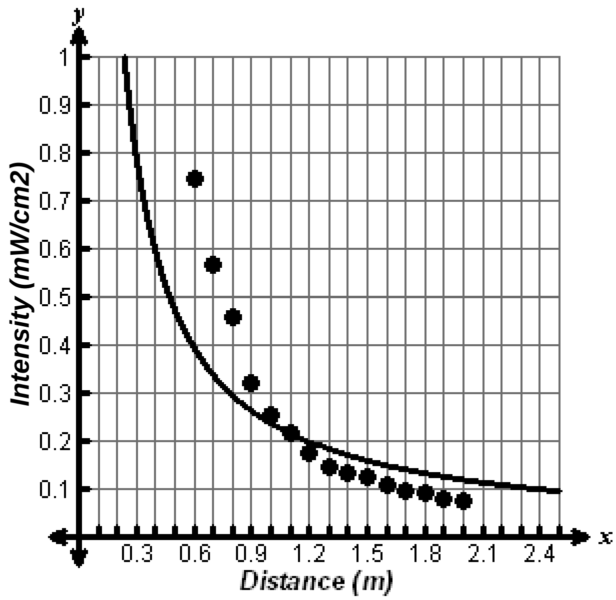


5. Substitute this value of *k* into the parent function and verify using a graph. From your Scatterplot, click the **Functions** button.

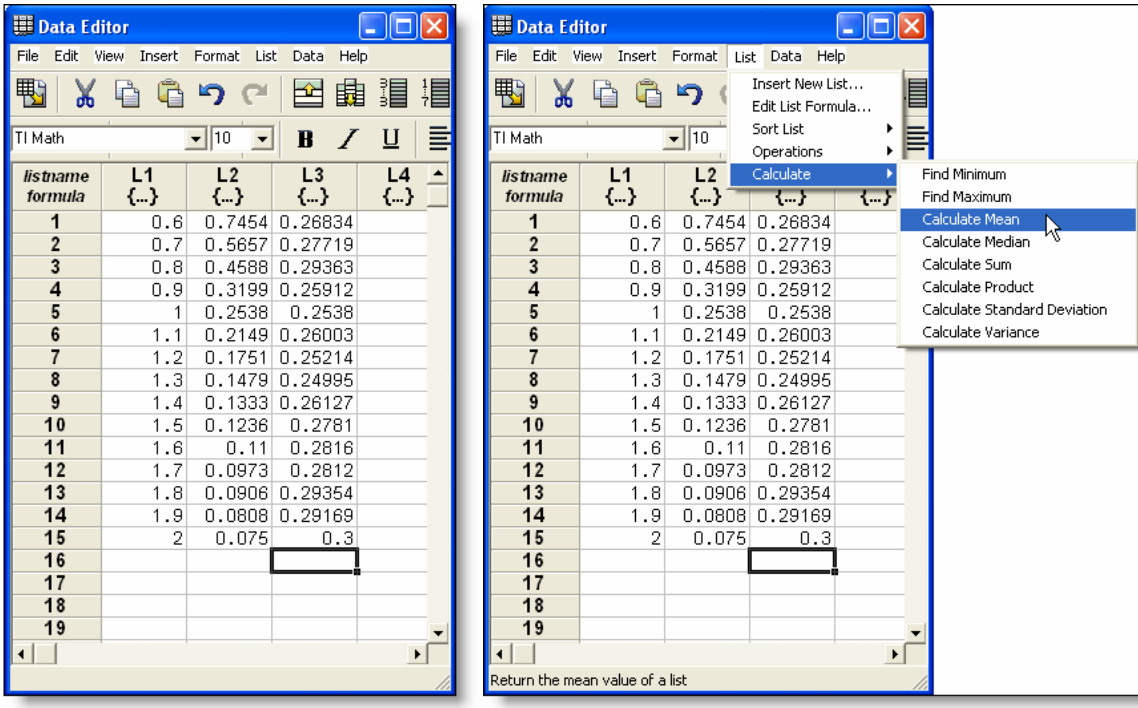


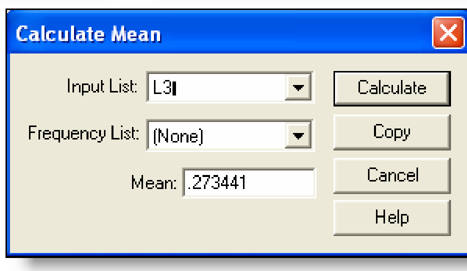
Inside the **Functions** dialog box, click the **f(x)** tab, then enter your function in the top text box. Click **Close** when complete.





6. This function is not a good fit. Try inverse-square variation, $y = \frac{k}{x^2}$. Multiply $x^2 y$ in order to find an approximate value for *k*, the constant of variation. In the **Data Editor**, clear **L3** then repeat Steps 1 through 5. Set **L3 = (L1)^2 × L2** by following steps 1 and 2. Find the average value of L3 by following Step 3.





Graph the function over the scatterplot, substituting the average value of L3 for *k*.